HF radar investigation of source terms in the Hasselmann equation

Stuart Anderson
University of Adelaide

2ND INTERNATIONAL WORKSHOP ON WAVES, STORM SURGES AND COASTAL HAZARDS

The spark that ignited my interest

" … the values of different wind input terms scatter by a factor of 300 – 500 % ([1], [2])"

from

Zakharov, V., Resio, D., and Pushkarev, A.: Balanced source terms for wave generation within the Hasselmann equation, Nonlin. Processes Geophys., 24, 581–597, https://doi.org/10.5194/npg-24-581-2017, 2017.

which referenced

[1] Badulin, S. I., Pushkarev, A. N., Resio, D., and Zakharov, V. E.: Self-similarity of wind-driven seas, Nonlin. Proc. Geoph., 12, 891–945, https://doi.org/10.5194/npg-12-891-2005, 2005.

[2] Pushkarev, A. and Zakharov, V.: Limited fetch revisited: comparison of wind input terms, in surface wave modeling, Ocean Model., 103, 18–37, https://doi.org/10.1016/j.ocemod.2016.03.005, 2016.

The spread of models for wind-wave growth :

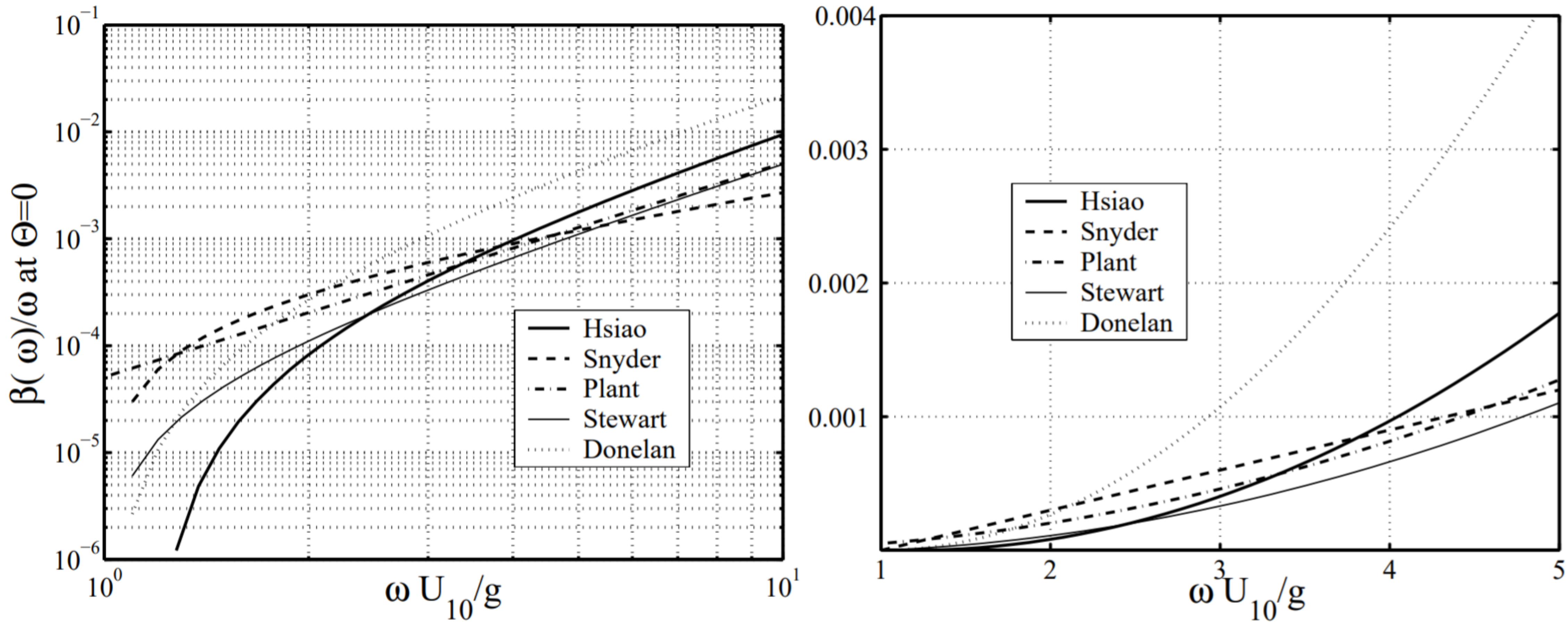


Fig. 1. Dependence of wind-wave growth rate on non-dimensional frequency $\omega U_{10}/g$ in log- and linear scales given by different experimental parameterizations (see legends).

S. I. Badulin, A. N. Pushkarev, D. Resio and V. E. Zakharov, 'Self-similarity of wind-driven seas', *Nonlinear Processes in Geophysics, 12, 891–945, 2005*

The Hasselmann equation I

The evolution of the wave field is often described by the action balance equation

$$\frac{\partial N}{\partial t} + \nabla_x \cdot (\nabla_\kappa \Omega N) - \nabla_\kappa \cdot (\nabla_x \Omega N) = S_{in} + S_{nl} + S_{dis}$$

$$= \quad 0$$

under steady state conditions

where the wave action density $N(\vec{\kappa})$ is related to the wave displacement spectrum $S(\vec{\kappa})$ by

$$N(\kappa) = \frac{\rho_w g S(\vec{\kappa})}{\sigma(\kappa)}$$

with σ the intrinsic frequency,

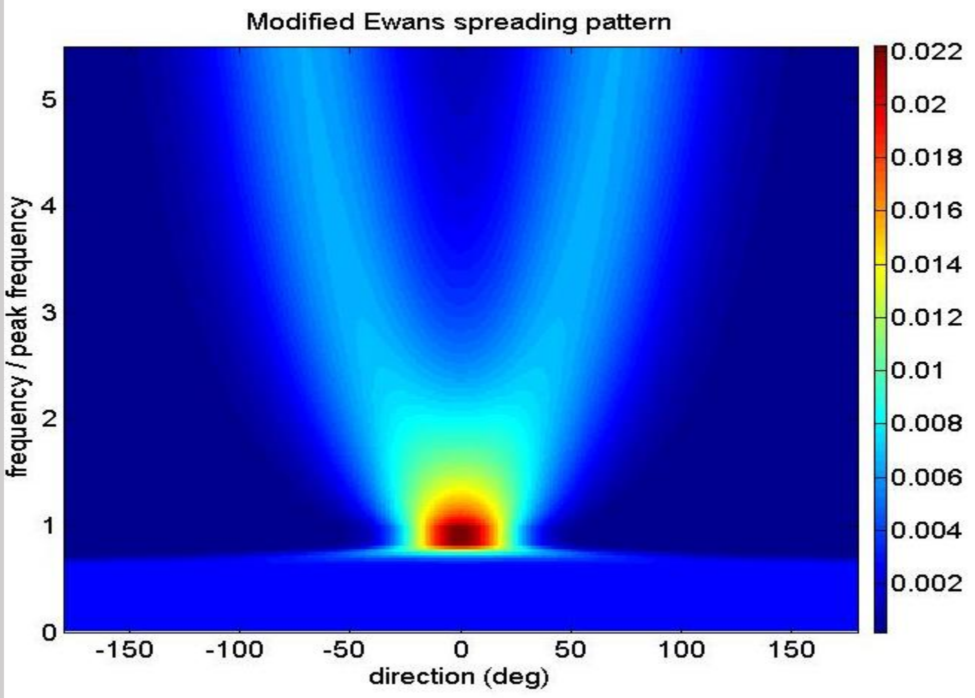
$$\Omega = \vec{\kappa} \cdot \vec{U} + \sigma$$

input from the wind

spectral flux due to nonlinear interactions

loss via dissipative processes

Steady state requires directional bimodality (Komen *et al,* 1984)



The Hasselmann equation II

For a wide range of conditions, the action balance equation reduces to the familiar form in terms of the energy spectral density - the Hasselmann equation :

$$\frac{\partial S(\vec{\kappa})}{\partial t} + \nabla_x \cdot \left(\nabla_\kappa \Omega S(\vec{\kappa})\right) = S_{in} + S_{nl} + S_{dis}$$

and it is this that forms the basis of the main wave modelling codes. There it is convenient to use frequency-angle coordinates,

$$\frac{1}{\rho_w g} N(\vec{\kappa}) d\vec{\kappa} = \frac{S(\vec{\kappa}) d\vec{\kappa}}{\sigma(\kappa)} = \frac{g^3}{2\sigma^4} S(\sigma, \varphi) d\sigma d\varphi$$

The challenge is to find mathematical models for the source terms S_{in}, S_{nl} and S_{dis}

Dissipation mechanisms for wind-generated surface gravity waves

$$\frac{\partial N}{\partial t} + \nabla_x \cdot \left(\nabla_\kappa \Omega N \right) - \nabla_\kappa \cdot \left(\nabla_x \Omega N \right) = S_{in} + S_{nl} + S_{dis}$$

$$S_{scat} + S_{frict} + S_{flex} + S_{visc}$$

$$S_{dis} = S_{wc} + S_{mv} + S_{tv} + S_{ma} + S_{wci} + S_{iw} + S_{bf} + S_{biwb} + S_{wiw} + S_{ice} + S_{BF}$$

white-capping

molecular viscosity

turbulent viscosity

Marangoni damping

wave-current interactions

internal wave coupling

bottom friction

bottom-induced wave breaking

wave-induced winds

sea ice coupling

Benjamin Feir instability / Fermi-Pastra-Ulam recurrence

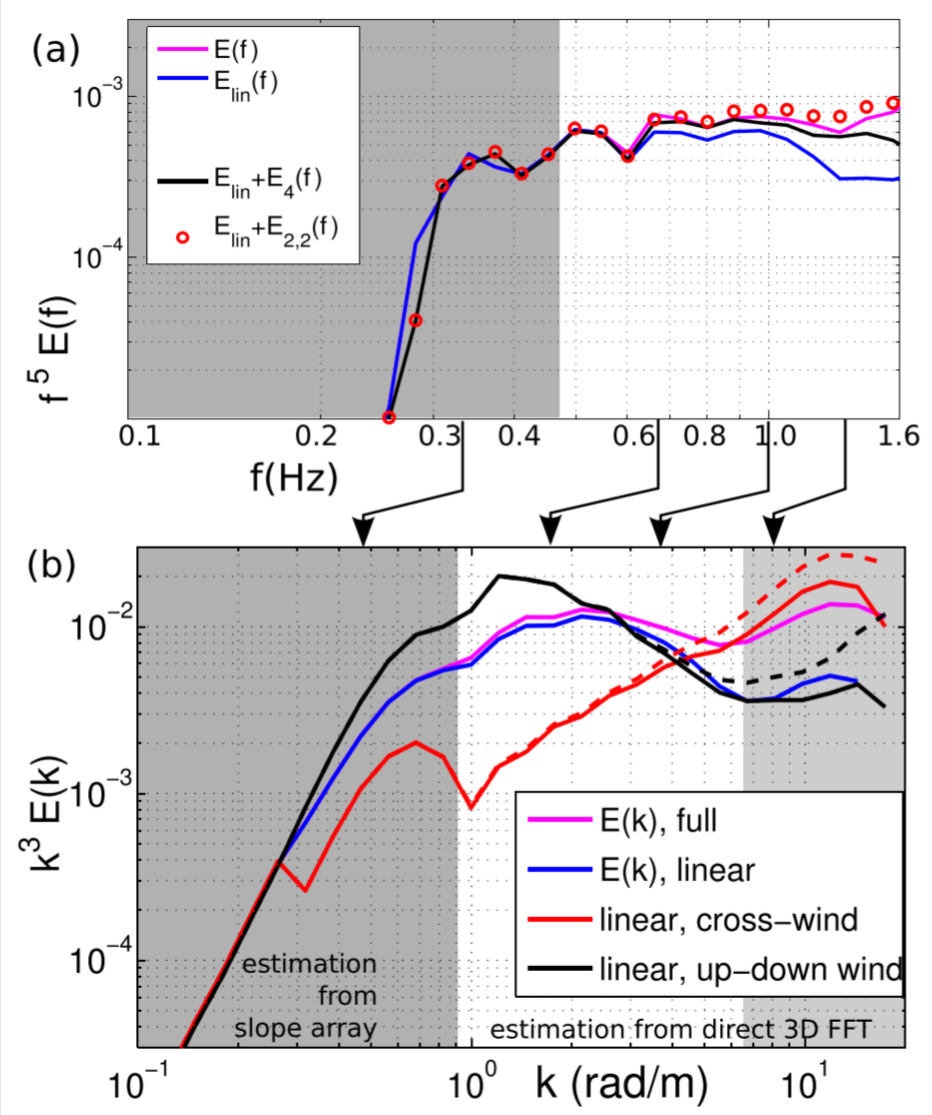
Wavenumber-direction space or frequency-direction space ?

Grid design and other considerations favour frequency-direction space for numerical wave modelling (eg SWAN, WAVEWATCH III)

From a radar perspective, wavenumber-direction space is better as the scattering integrals have an elegant physical interpretation in terms of mulltiple Bragg (resoant) scattering

The Jacobian is not ill-behaved, so both forms have been used by the HF radar community

There is a nice paper on the transformation by Hsu *et al*, in *China Ocean Eng*., Vol. 25, No. 1, pp. 133 – 138, 2011



See also F. Leckler, F. Ardhuin, C. Peureux, A. Benetazzo, F. Bergamasco, and V. Dulov, 'Analysis and Interpretation of Frequency–Wavenumber Spectra of Young Wind Waves', JPO, vol.45, pp. 2484-2496 from which the figure is taken

The SPM2 mapping from directional wave spectrum to radar Doppler spectrum

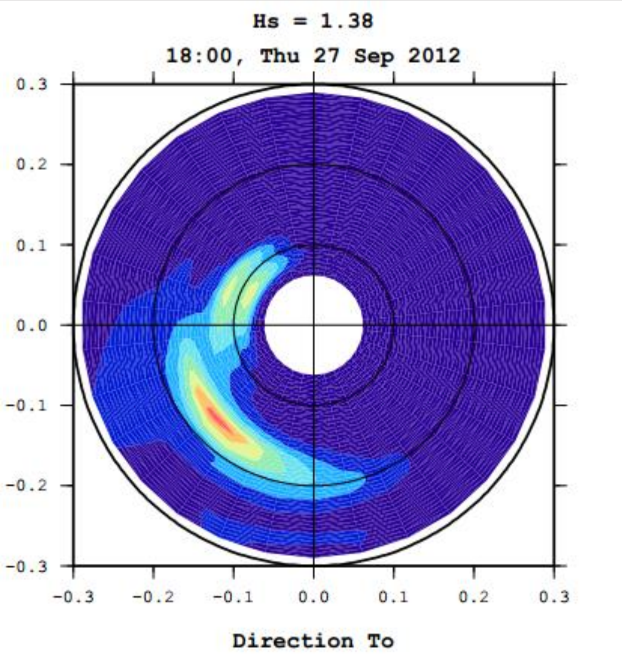
$$\tilde{S} = \int d\vec{k}\, \tilde{R}\left(\vec{k}', \vec{k}\right) \delta(\vec{k}' - \vec{k} + 2\vec{k}.\hat{n} \quad \hat{n})(.) +$$

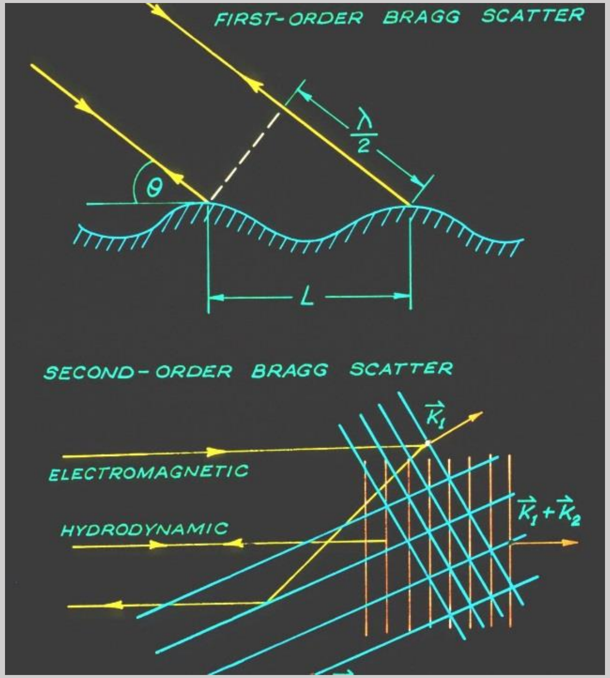
$$+\int d\vec{\kappa}_1 F^{(1)}(\vec{k}', \vec{k}, \vec{\kappa}_1) S(\vec{\kappa}_1)(.) +$$

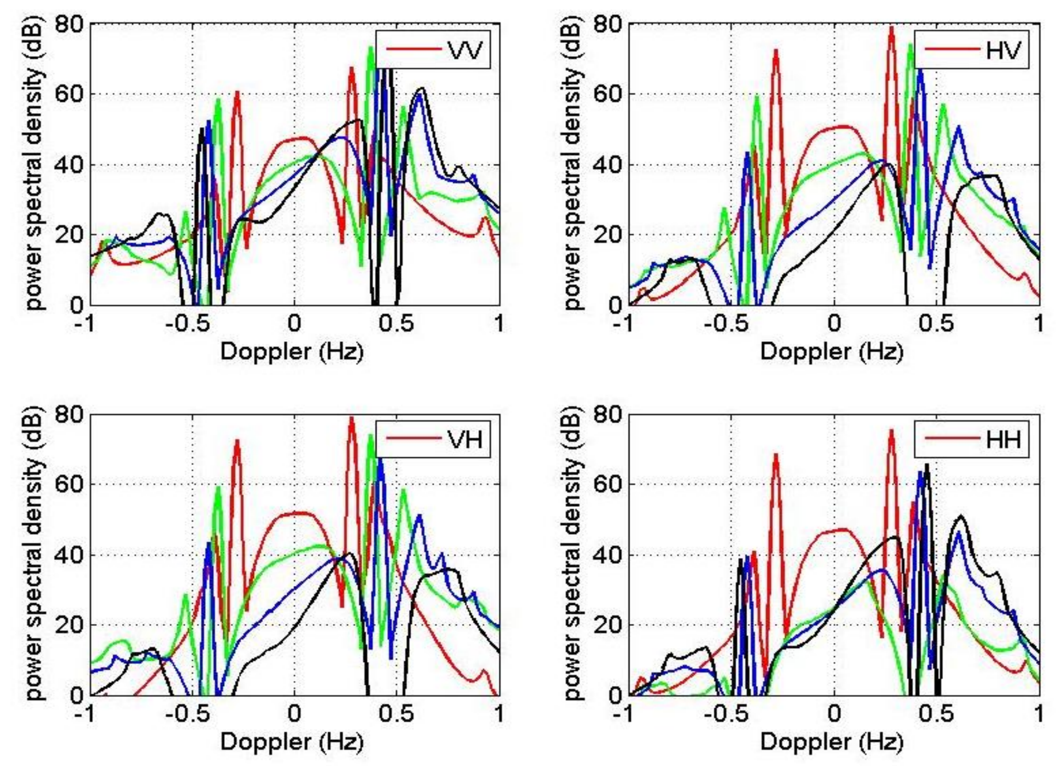
$$+\int\int d\vec{\kappa}_1 d\vec{\kappa}_2 F^{(2)}(\vec{k}', \vec{k}, \vec{\kappa}_1, \vec{\kappa}_2) S(\vec{\kappa}_1) S(\vec{\kappa}_2)(.) +$$

$$+.....$$

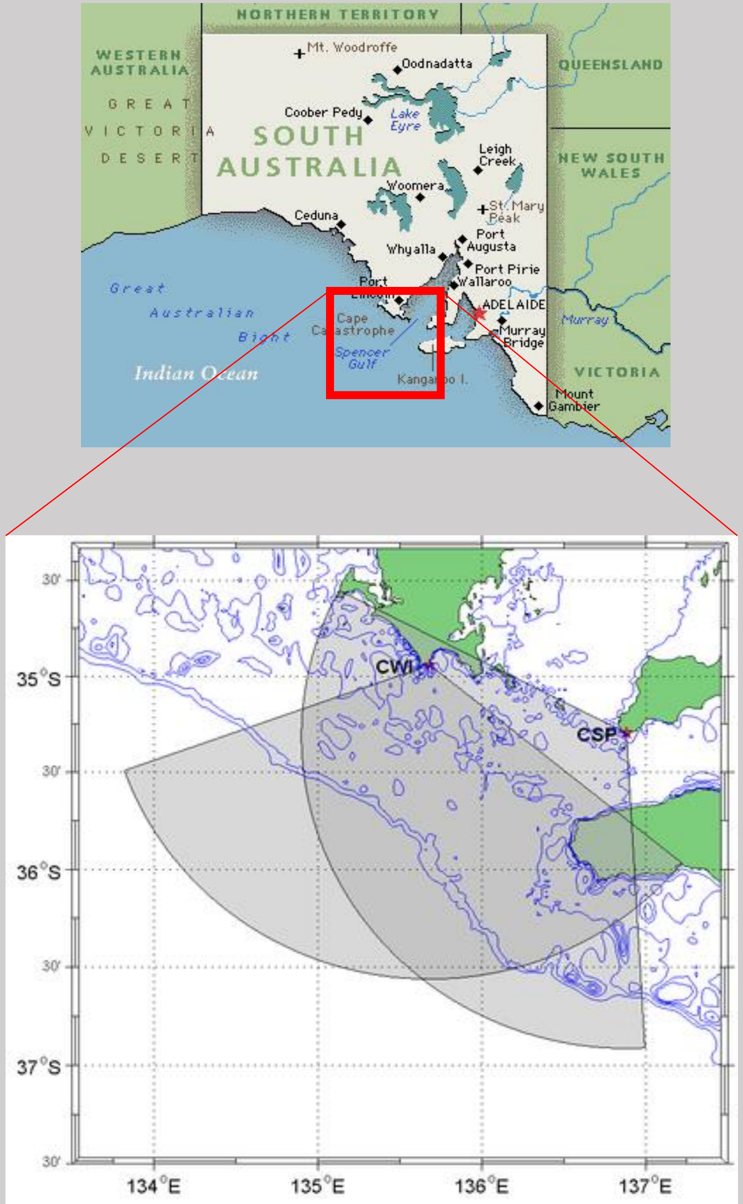
NB : incomplete 4th order contributions for non-Gaussian surfaces

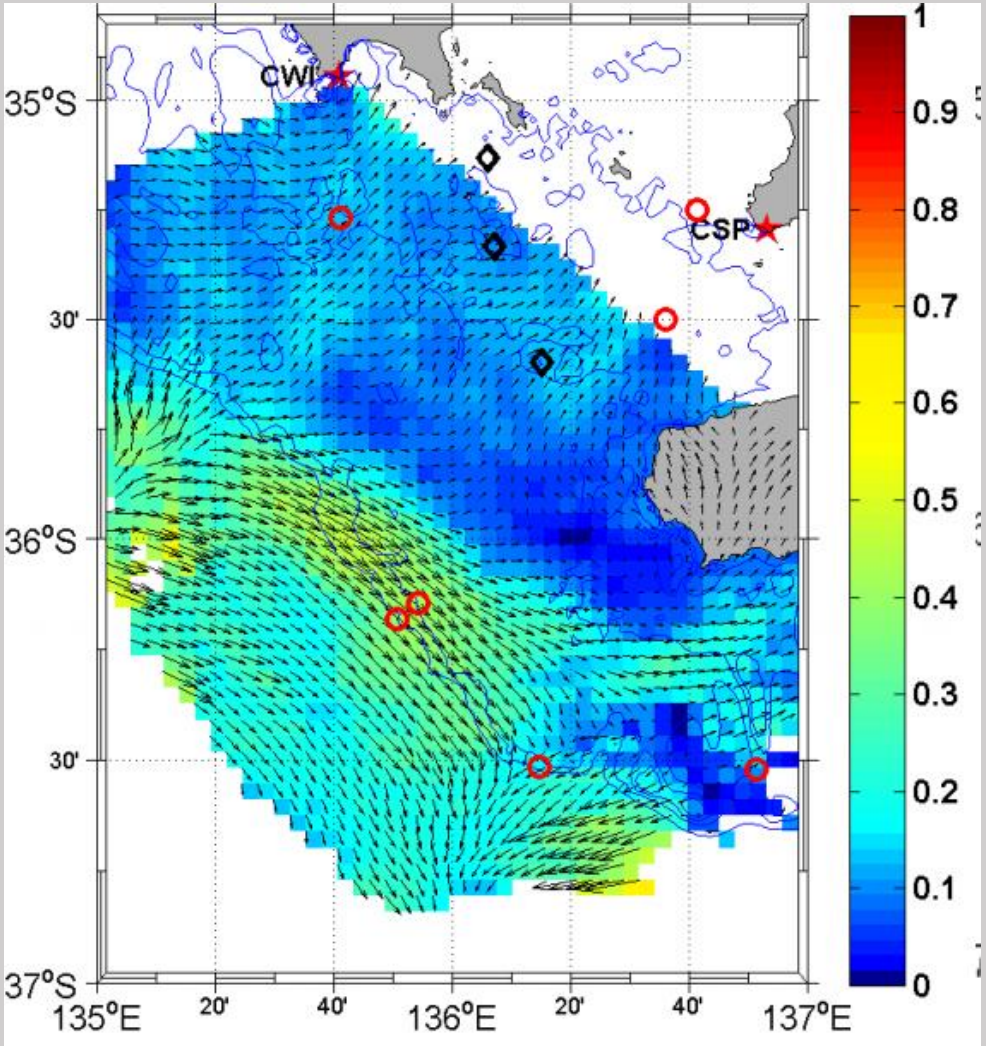




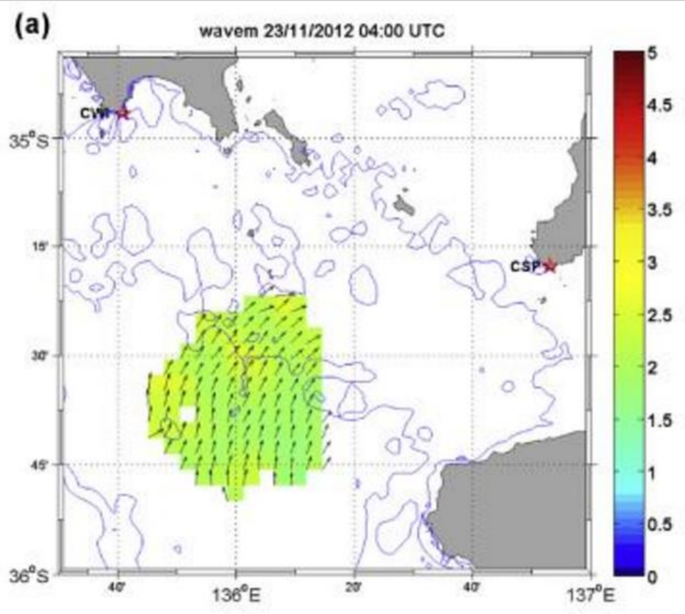


Currents, wind and wave information from the ACORN SA Gulf radar system

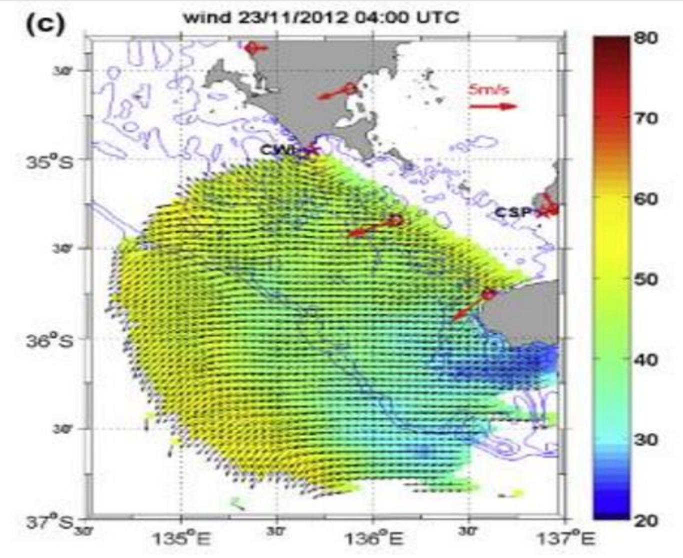




current velocity



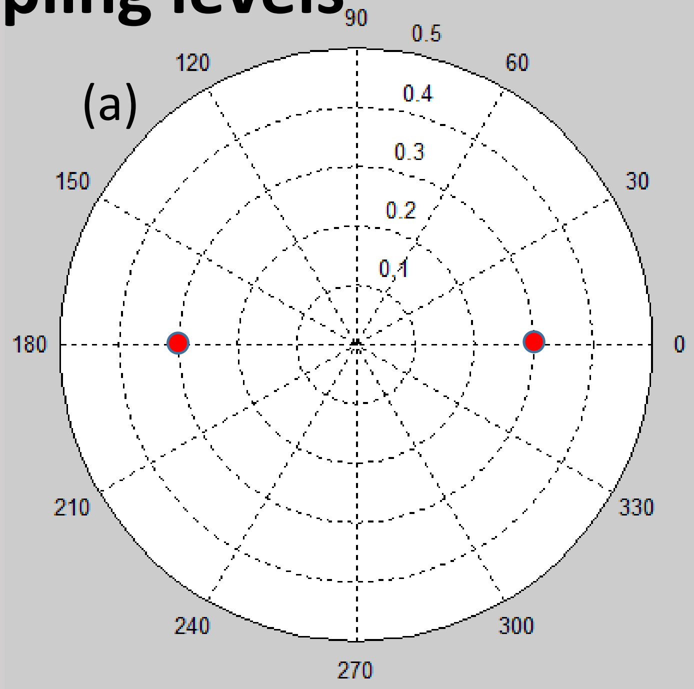
significant waveheight

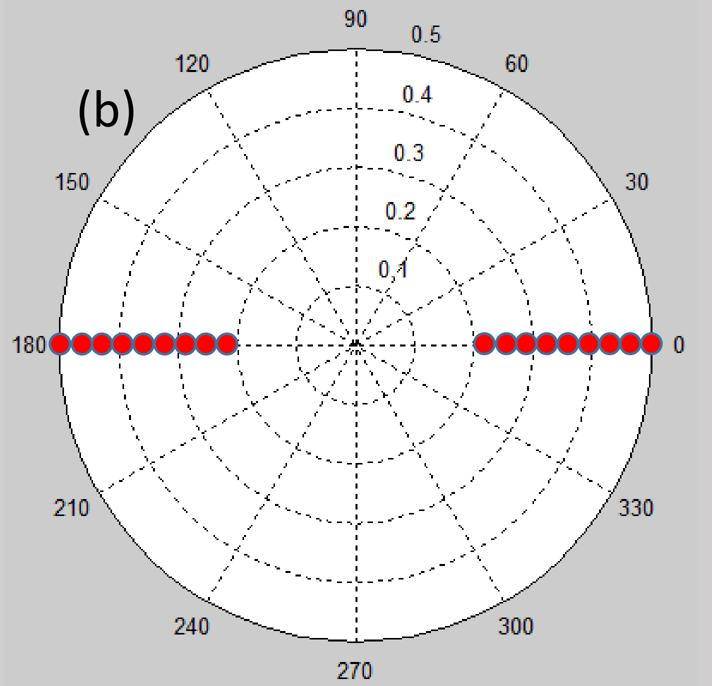


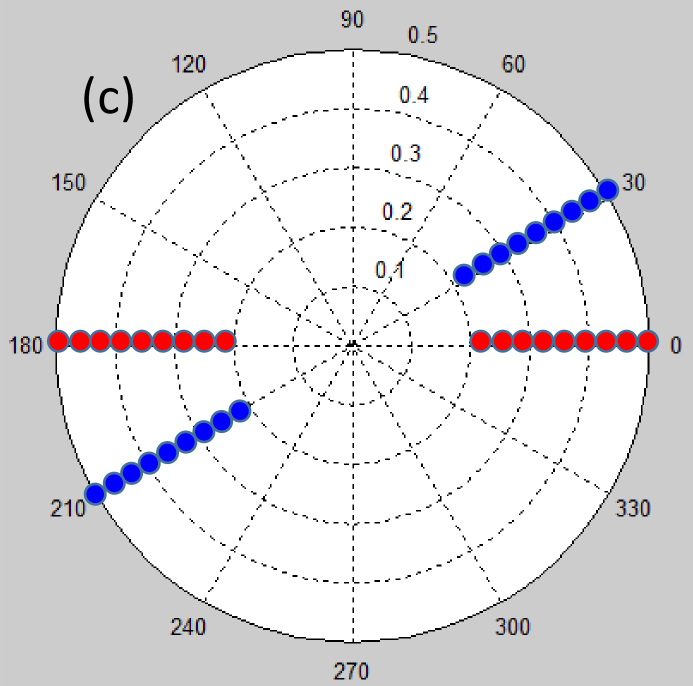
inferred surface wind

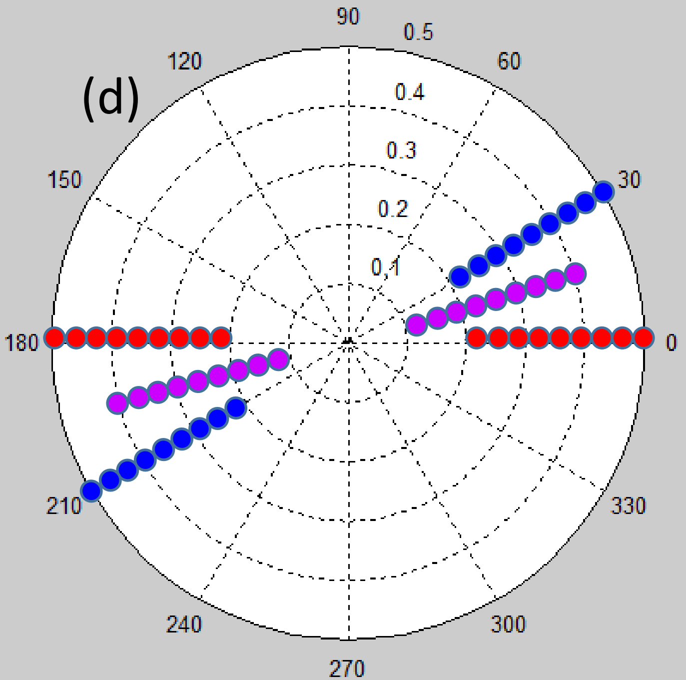
Directional wave spectrum sampling levels

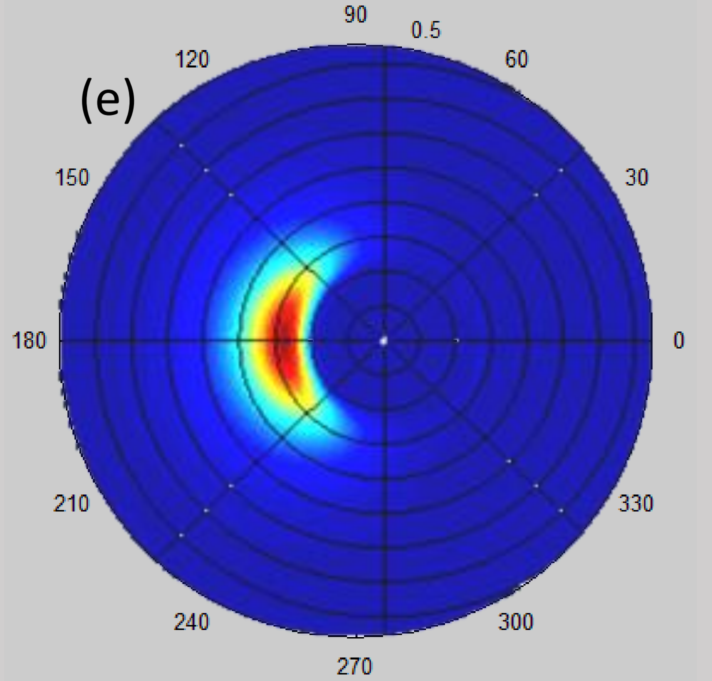
- Any HF radar can deliver (a)
- Multi-frequency radars can also do (b)
- If two radars illuminate the same patch of ocean, they can deliver (c)
- If bistatic mode is enabled, they can do (d)
- If the signal is uncorrupted and inversion is enabled, a single radar can do (e)











How might HF radar contribute to refining the source term models ?

What we have to offer is the ability to monitor properties of the directional wave spectrum on kilometre scale resolution, over large areas (10^4 – 10^6 sq km), with a refresh rate of order 100 – 1000 s

We do this by interacting directly with the surface gravity waves, not indirectly via capillary waves

We have developed libraries of radar signatures of quite a number of ocean and atmospheric phenomena, some validated by experiment, others awaiting opportunities to put them to the test

We recognise that solving an equation of the form

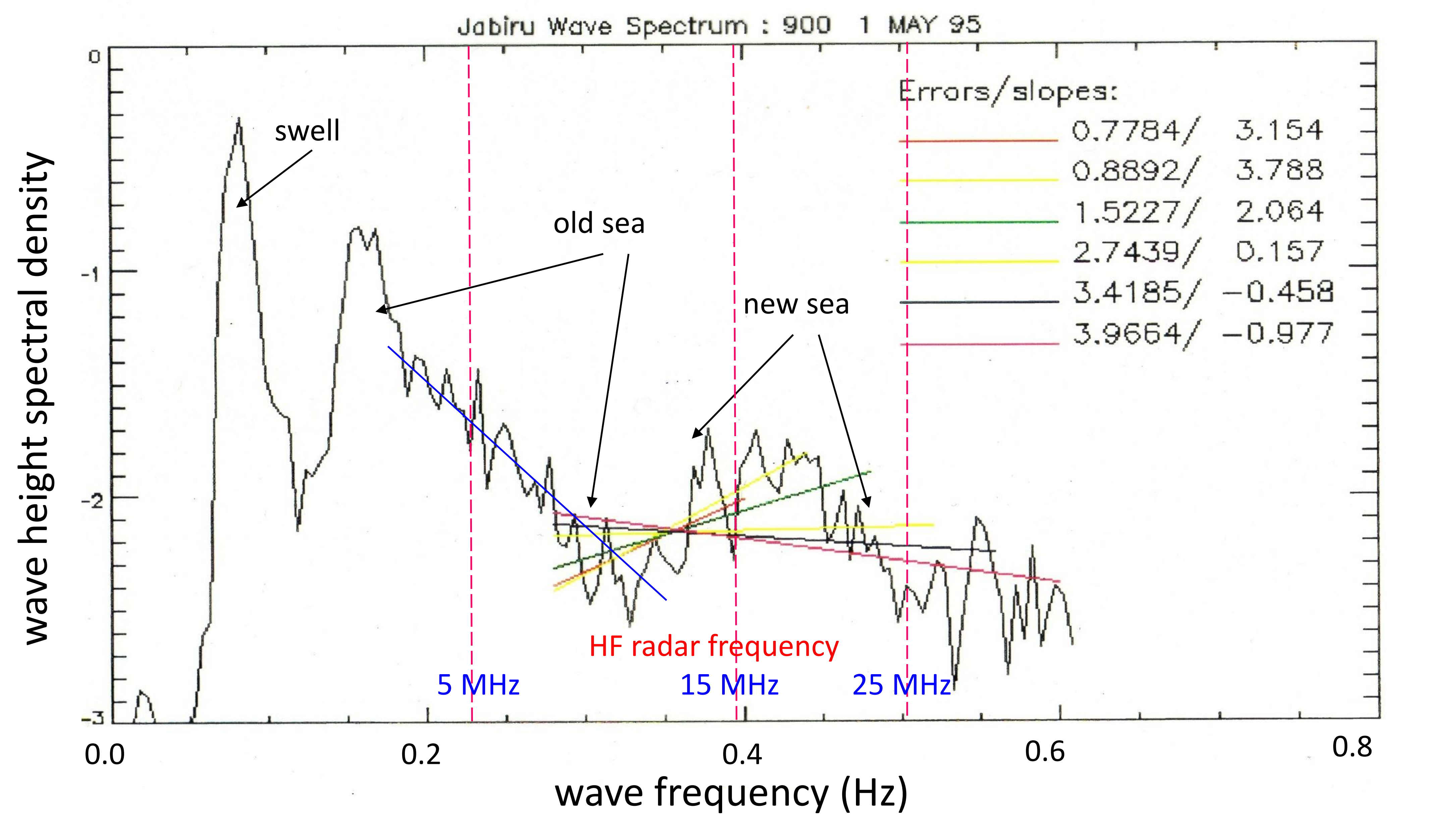
THE ***BRICKS WITHOUT STRAW*** CHALLENGE

$X + Y + Z = Q$

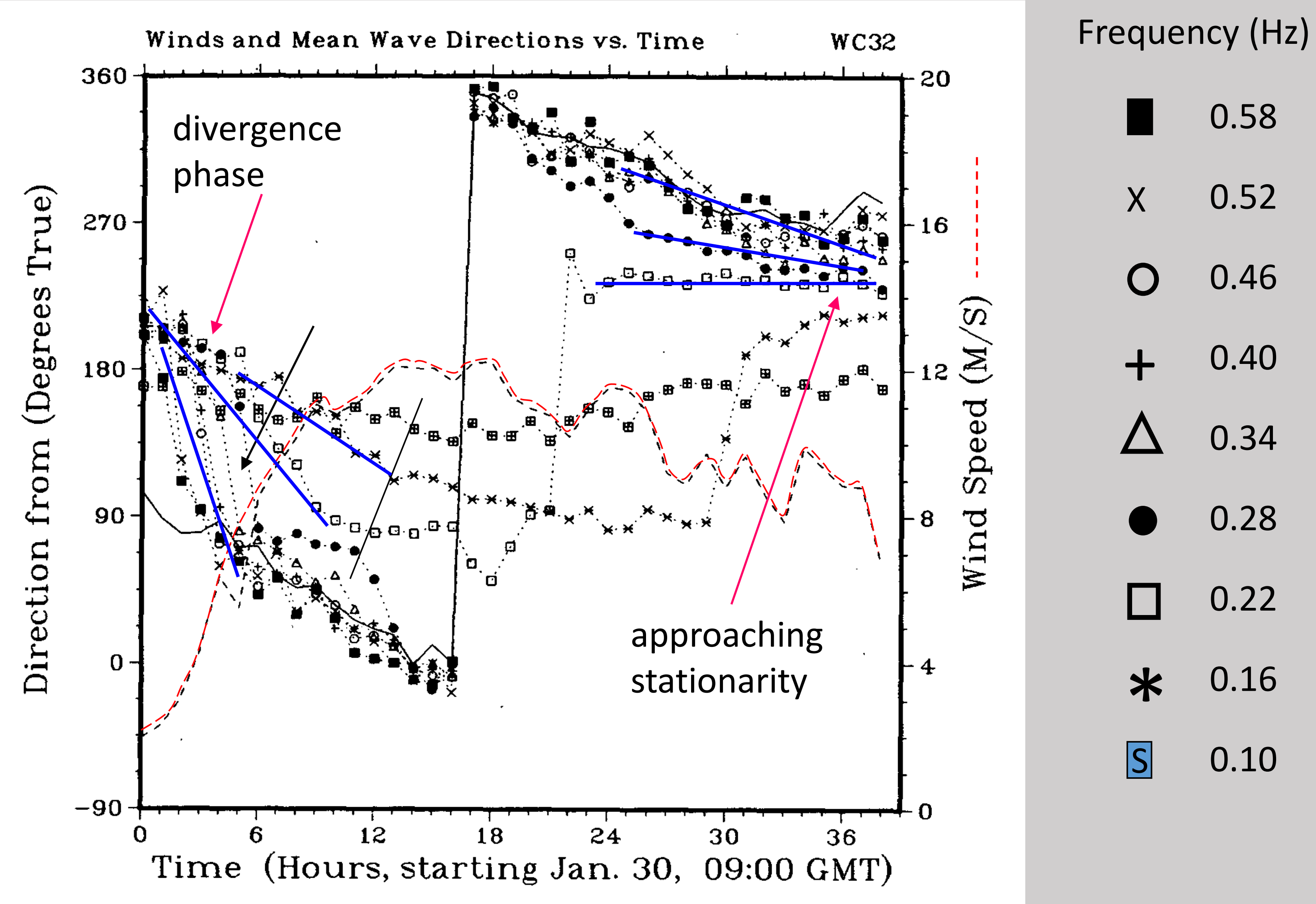
for the functions X, Y and Z, given only measurements of function Q sounds implausible if not impossible, but hope springs from several considerations :

- X is relatively well understood, though not easy to evaluate
- The equation plays out in the arena of ***space*** *and* ***time*** *and* ***wavenumber*** *and* ***direction***, which we can sample with resolution comparable with or better than the characteristic spatial and temporal scales of X, Y and Z
- There may be particular conditions in which either Y or Z may be assumed to dominate the other, setting aside X
- There may be particular conditions in which either Y or Z may vary much more rapidly than the other, setting aside X
- Perhaps conditions far from equilibrium will stress-test the equation

Representative omni-directional spectral structure of a non-stationary sea and HF radar Bragg frequencies

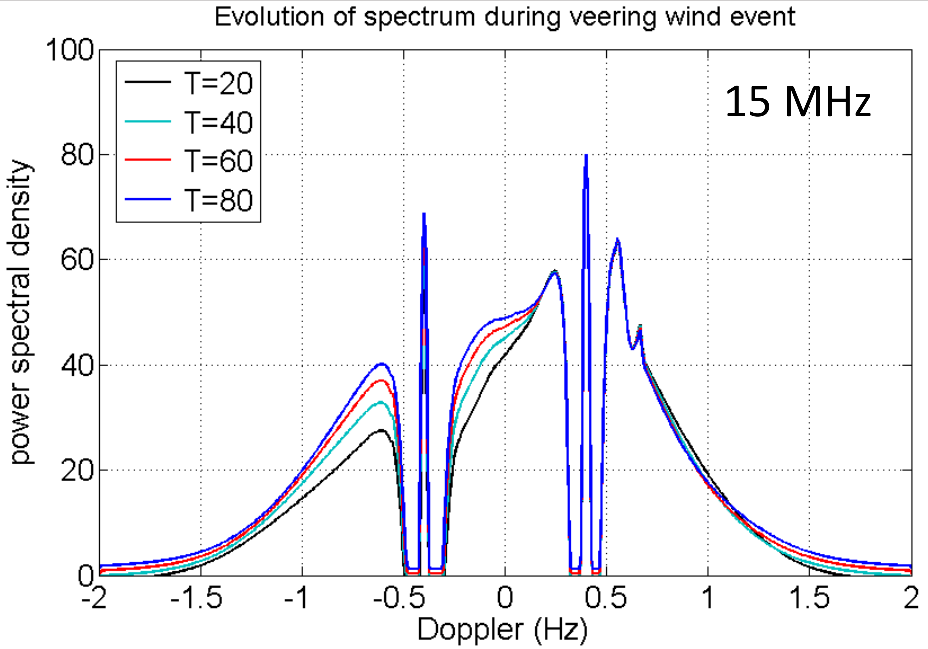


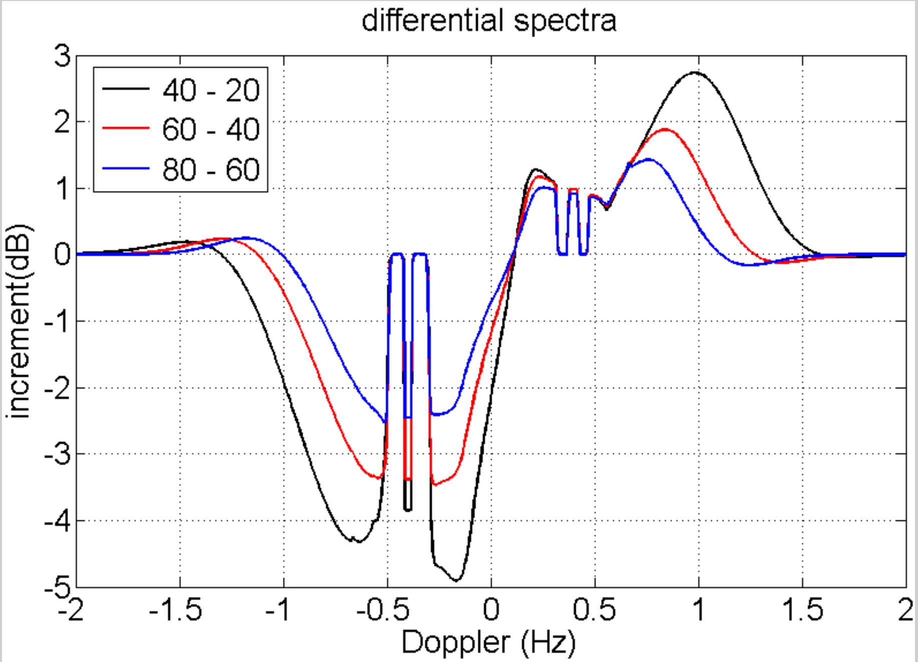
Measured directional response of a wave field under a progressive change in wind direction

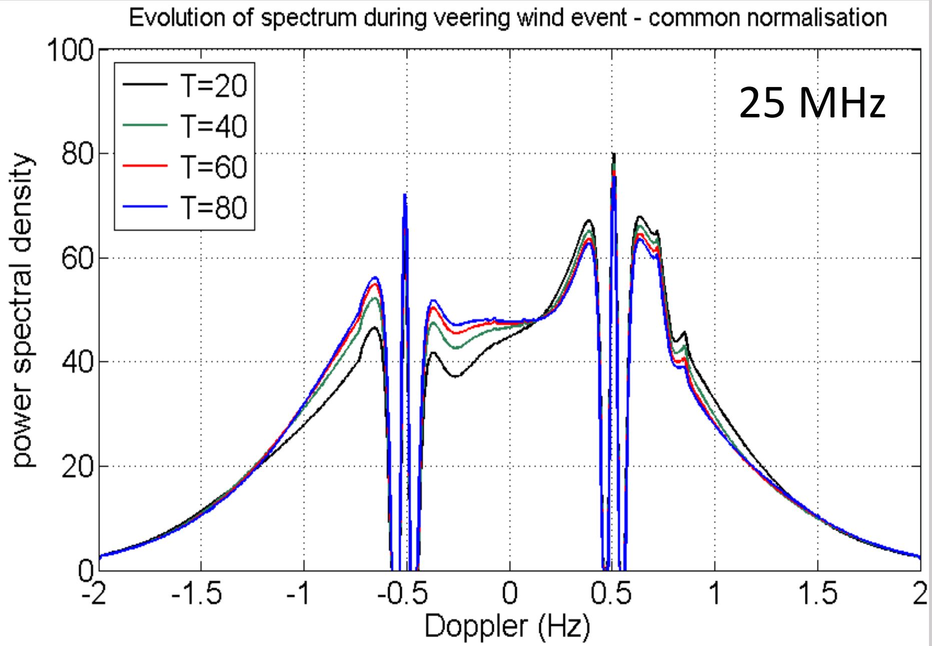


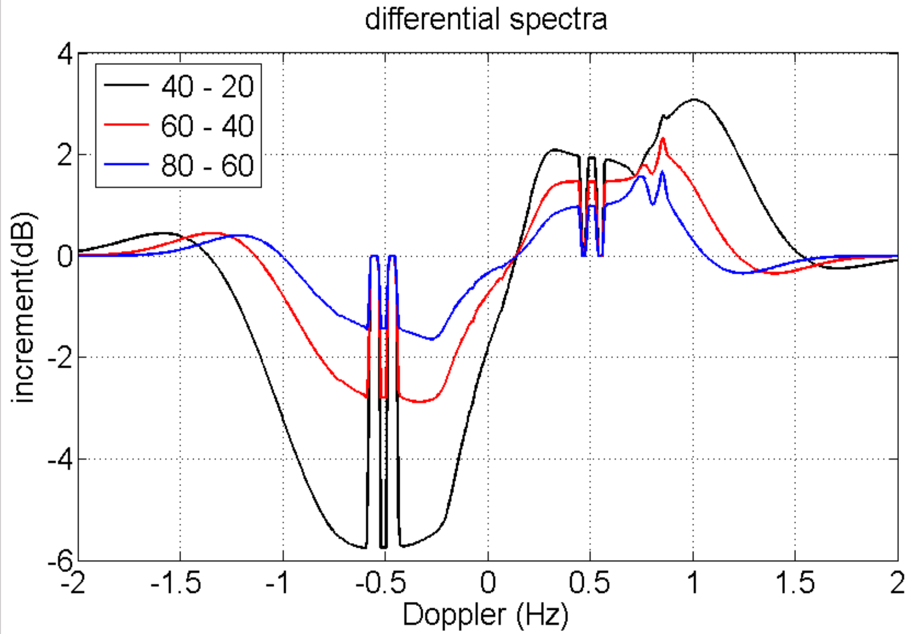
Anderson, 2012; adapted from Perrie and Toulany, 1995

Modelling examples : veering wind effects over 80 minutes

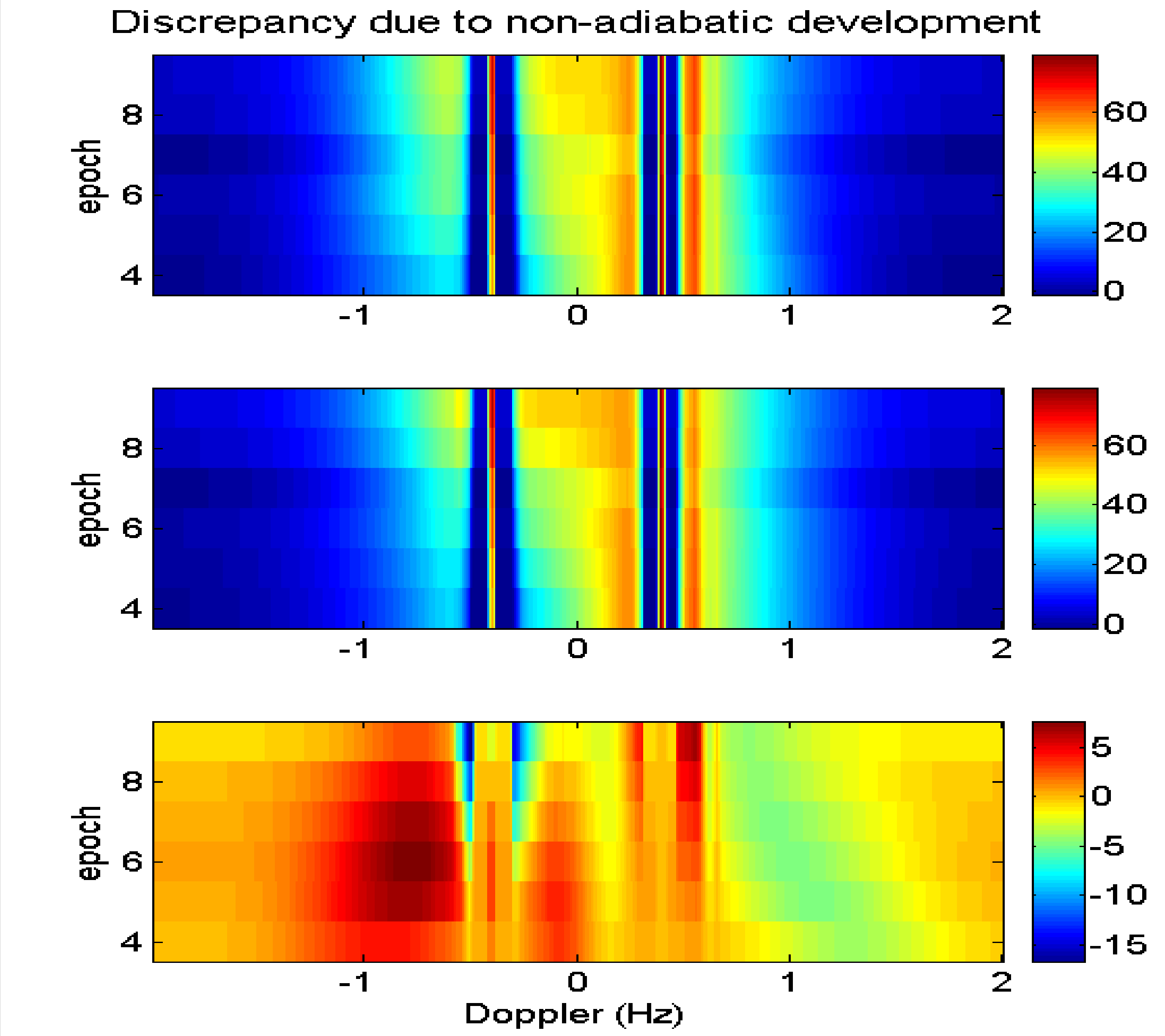








Discrepancy introduced by assuming adiabatic development

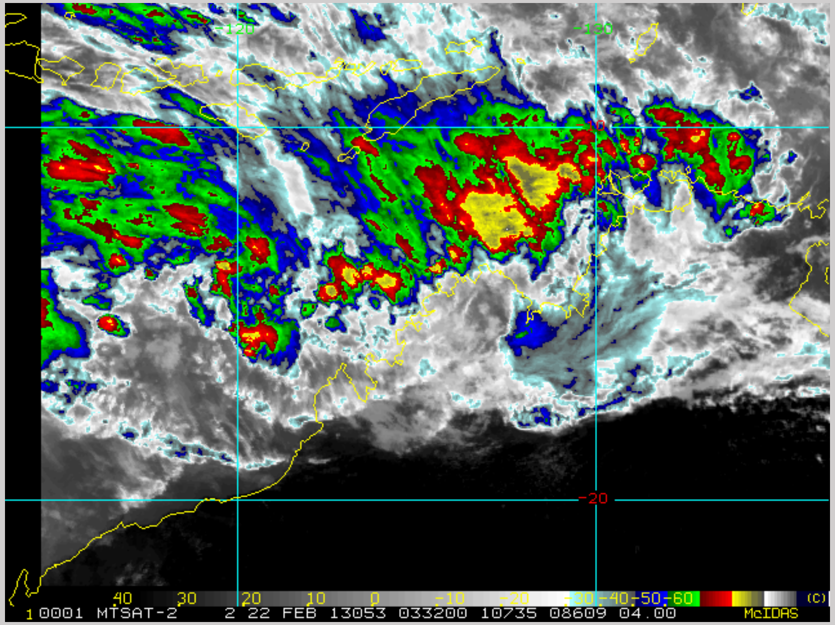


Doppler spectrum during early stage of evolution under veering wind

Doppler spectrum during early stage of evolution assuming adiabatic development

Discrepancy

Target observable #15 : Tropical convective cells



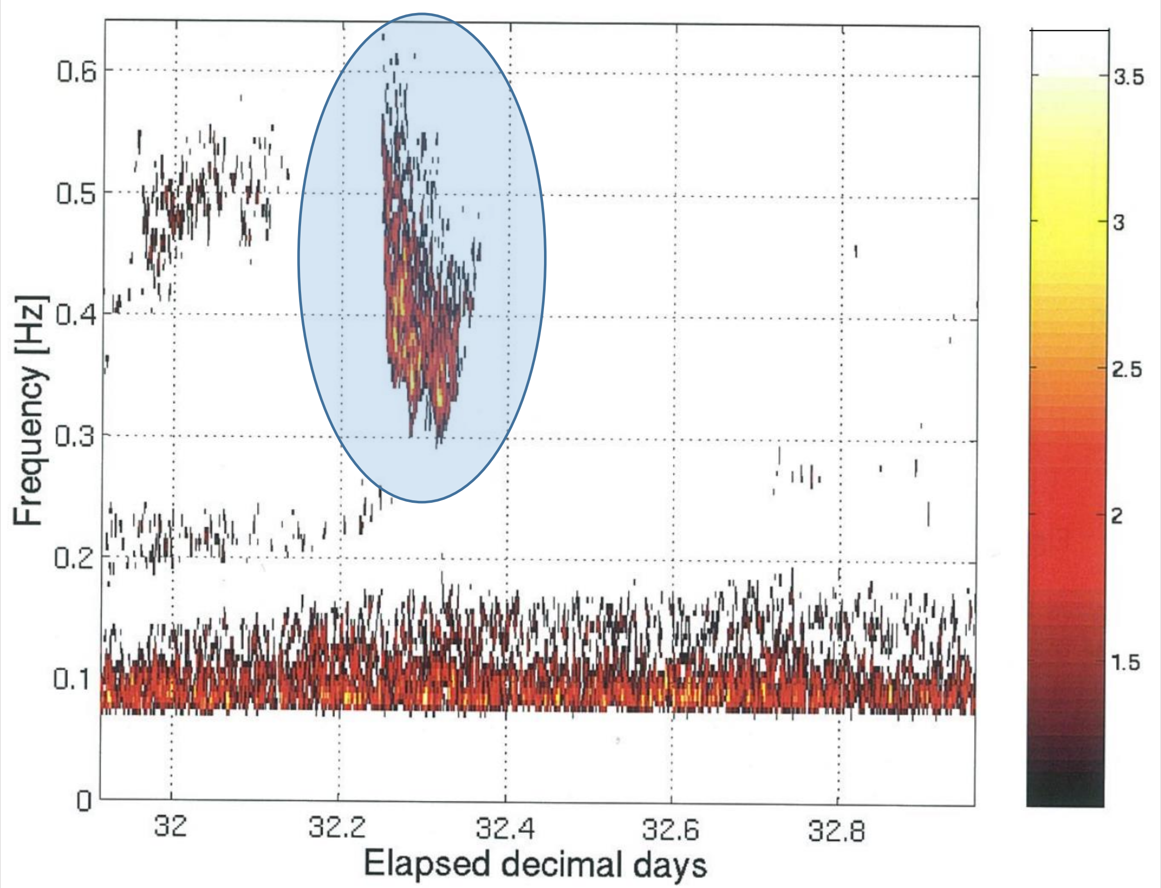
Sea state variation at a squall line

Wind speed ~ 20 m/s gusting to 25 m/s

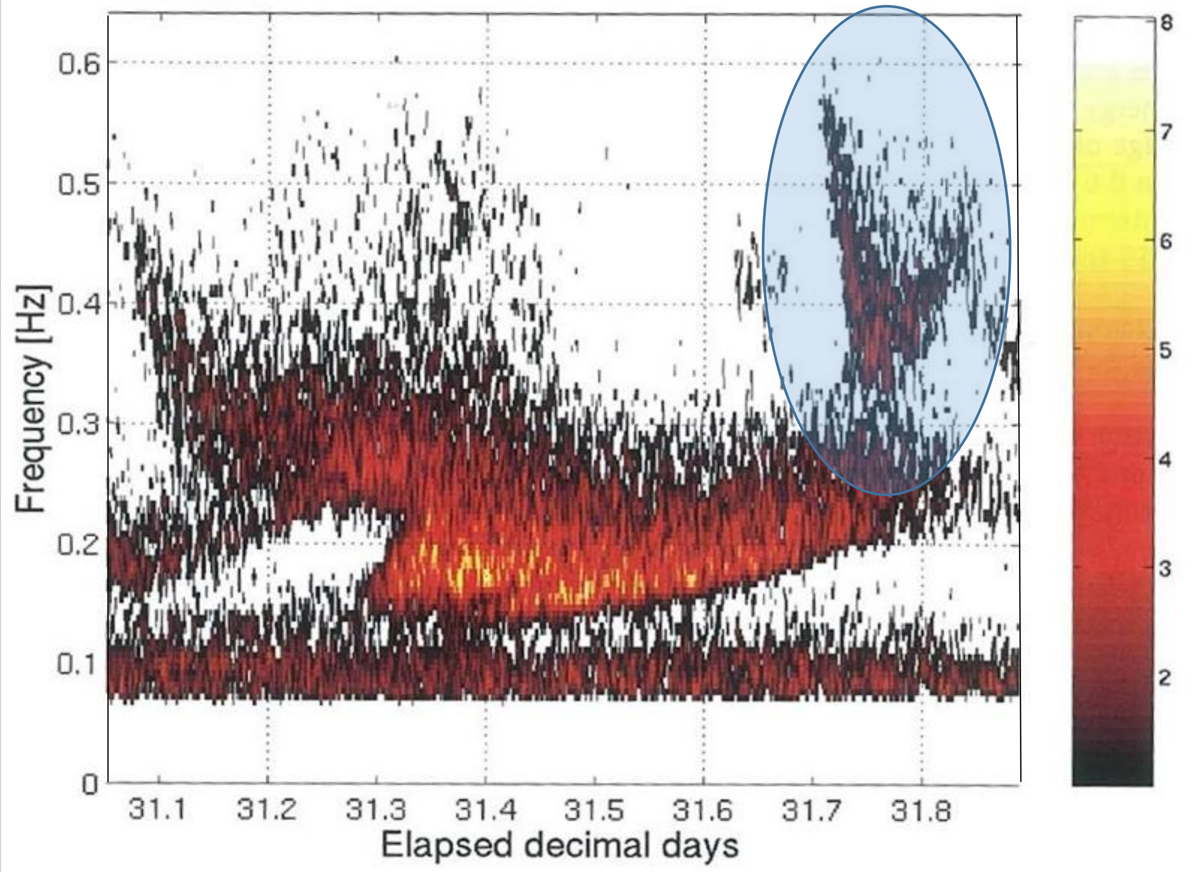
Roswold, 2010

Short waves excited by convective cells

NB   figures have different scales



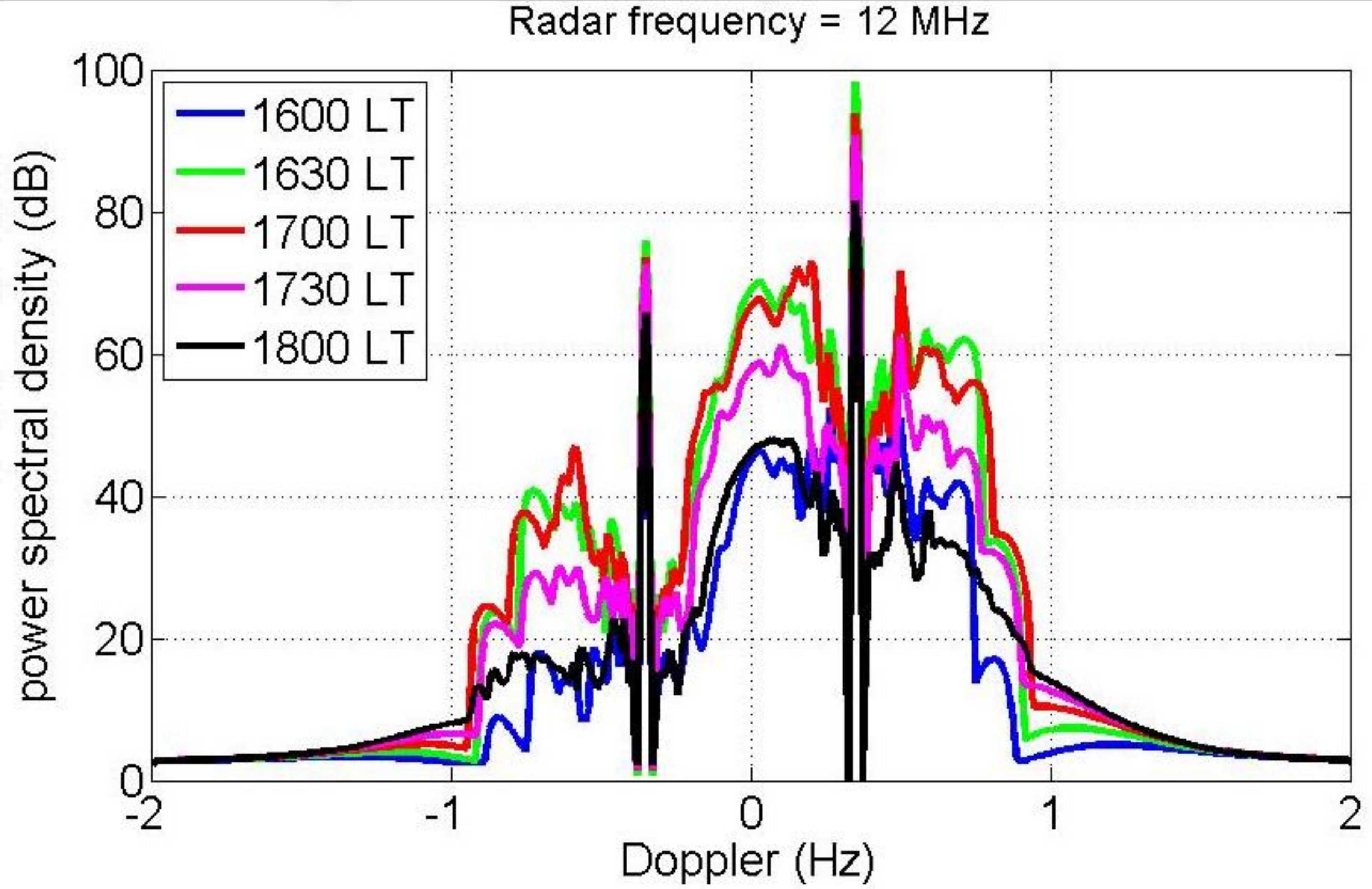
Convective disturbance in calm conditions   95:342



Convective disturbance occurring as longer waves decay following a brief storm

Schulz, 1995

Modelled HF radar Doppler spectrum evolution during the lifetime of the convective cell computed from measured wave spectra

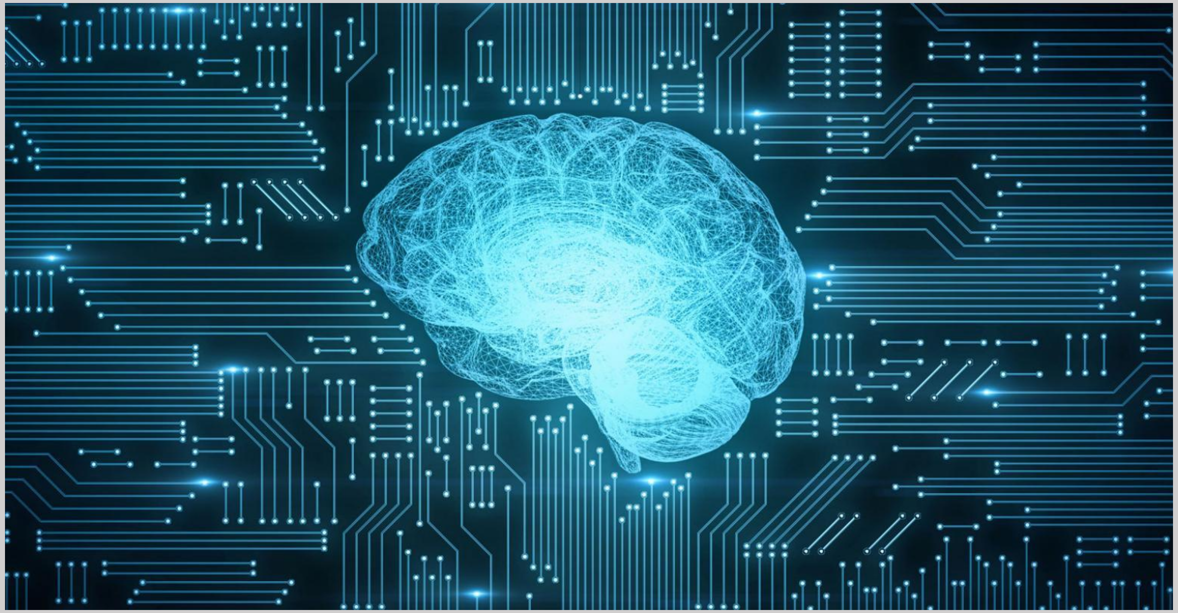


What other options do we have to help unravel the Hasselmann equation ?

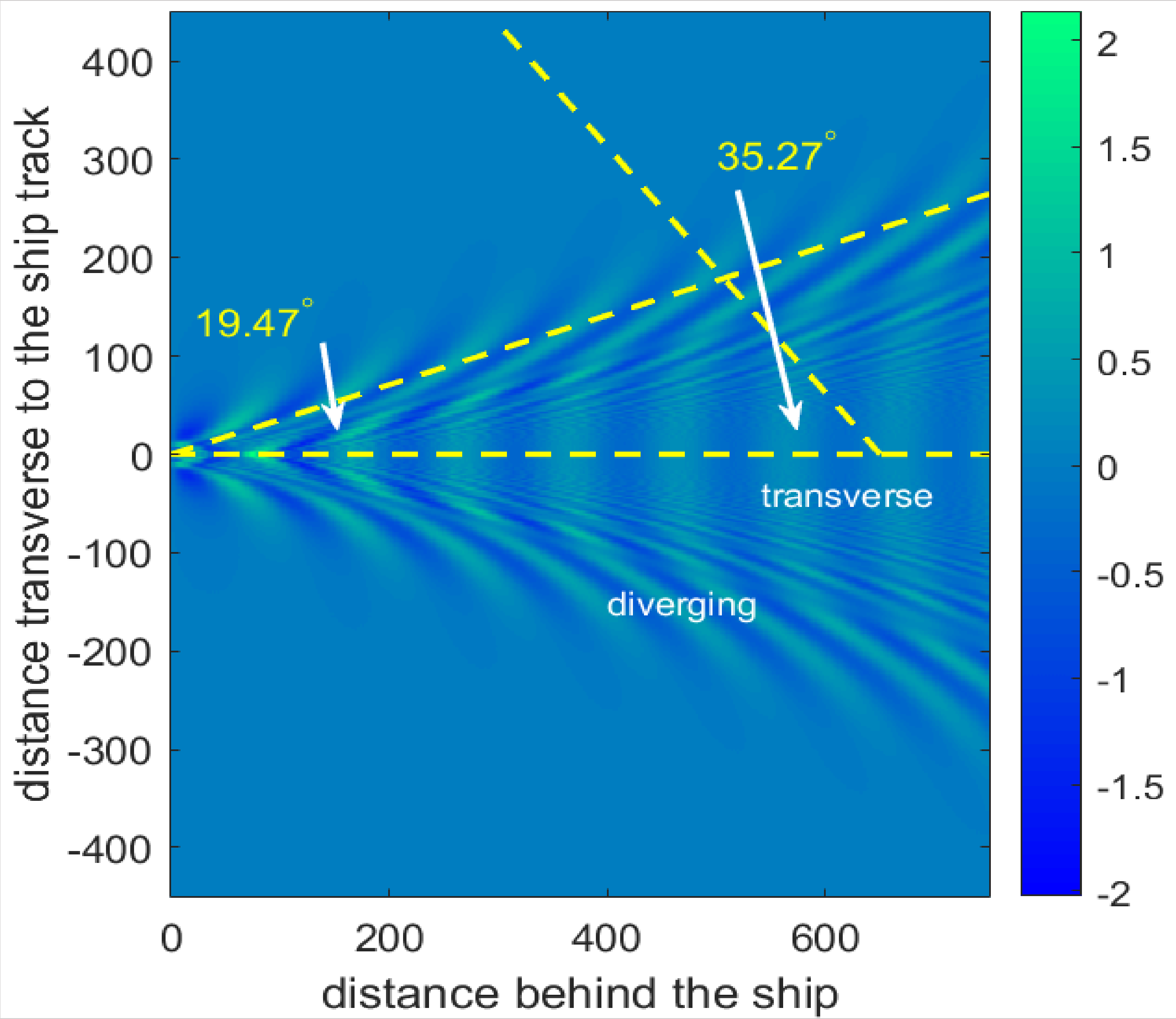
When Nature doesn't give us the waves we want, **MAKE THEM !**

When Nature's standard boundary conditions at the sea surface are not conducive to our study, **FIND OTHERS !**

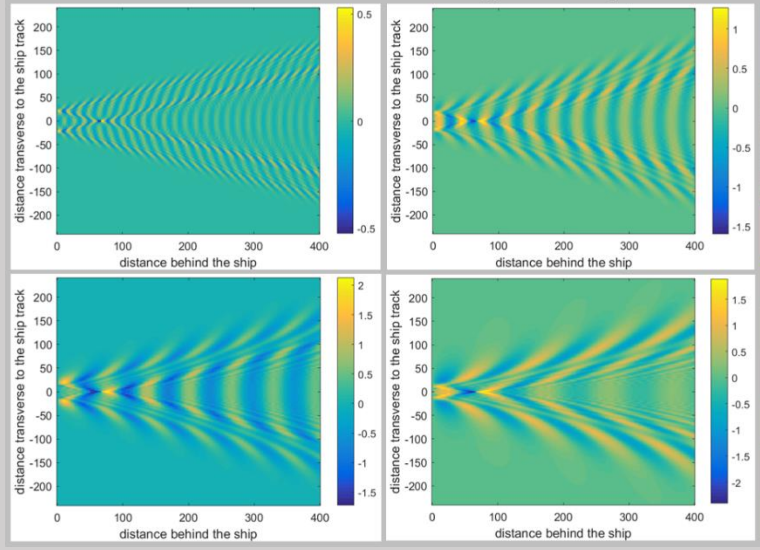
When it's all too hard for humans, **ASK A COMPUTER !**



Generic form of the Kelvin wake and computed wake patterns for two frigates

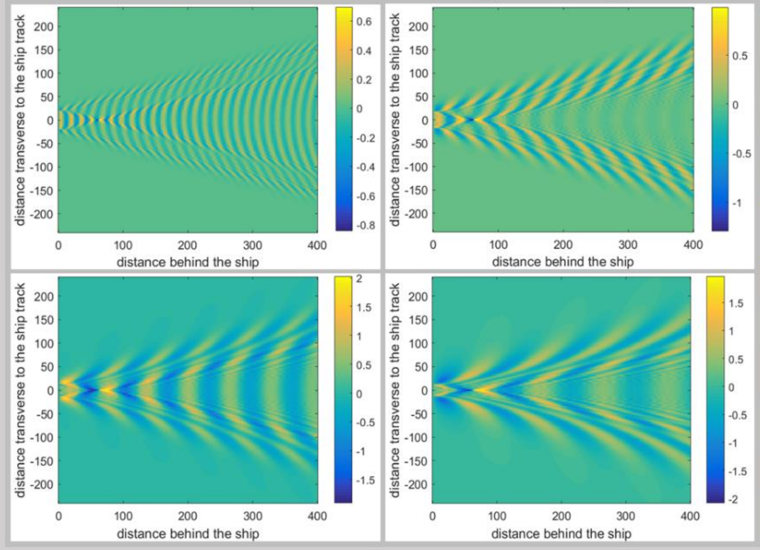


Modelled wake of Guided Missile Frigate at 10, 15 20 and 25 knots



SHIP A	
Displacement	4,200 tonnes
Length	138.1 metres
Beam	14.3 metres
Draught	7.5 metres
Power	31,000 kW
Speed	30 knots

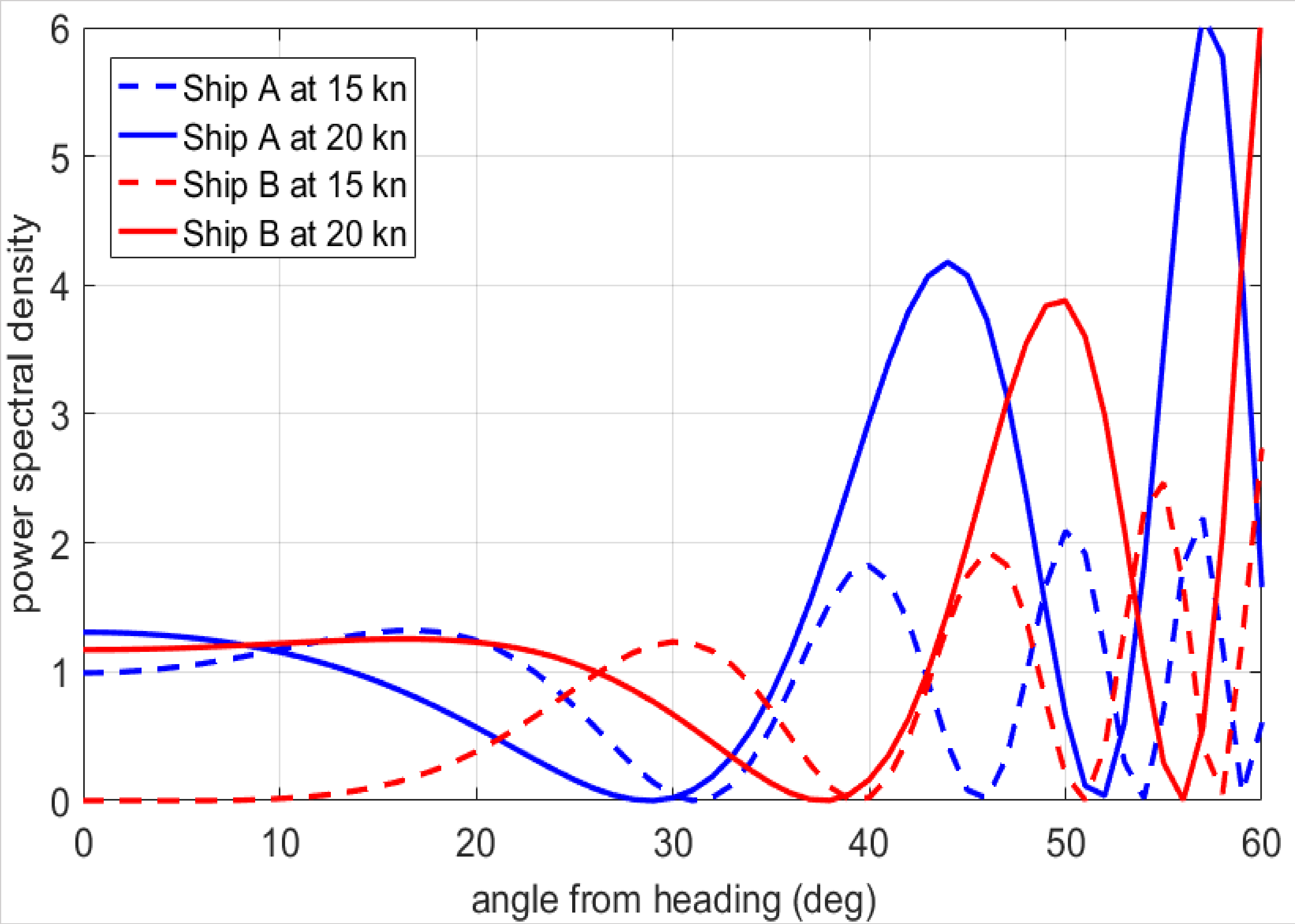
Modelled wake of Guided Missile Frigate at 10, 15 20 and 25 knots



SHIP B	
Displacement	4,500 tonnes
Length	124.8 metres
Beam	15.2 metres
Draught	4.2 metres
Power	45,400 kW
Speed	30 knots

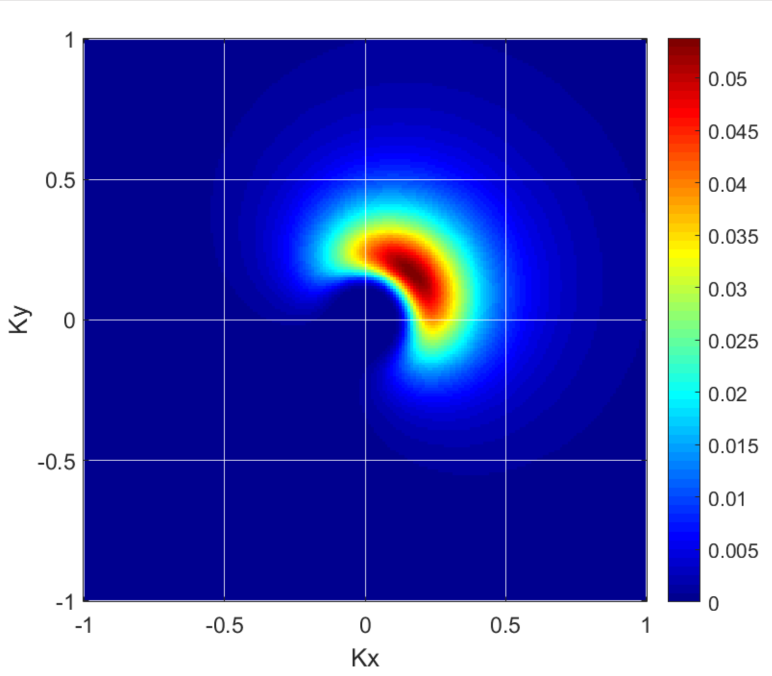
S. J. Anderson, 'HF radar signatures of ship wakes', Proceedings of the Progress in Electromagnetics Research Symposium, Singapore, October 2017

Comparison of angular spectra for two frigates, at two speeds



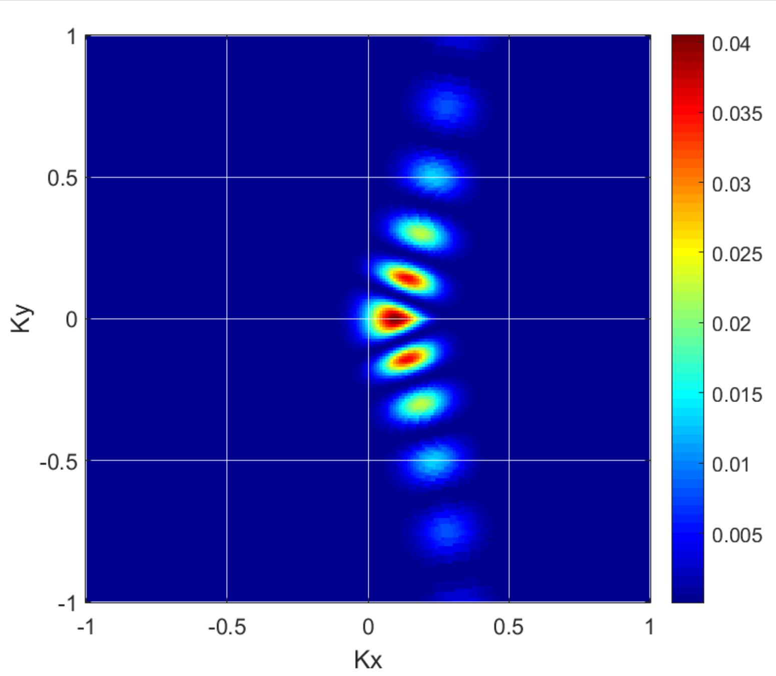
Strong prospect of discrimination capability

Surface wave spectra for the three components of the scattering integral



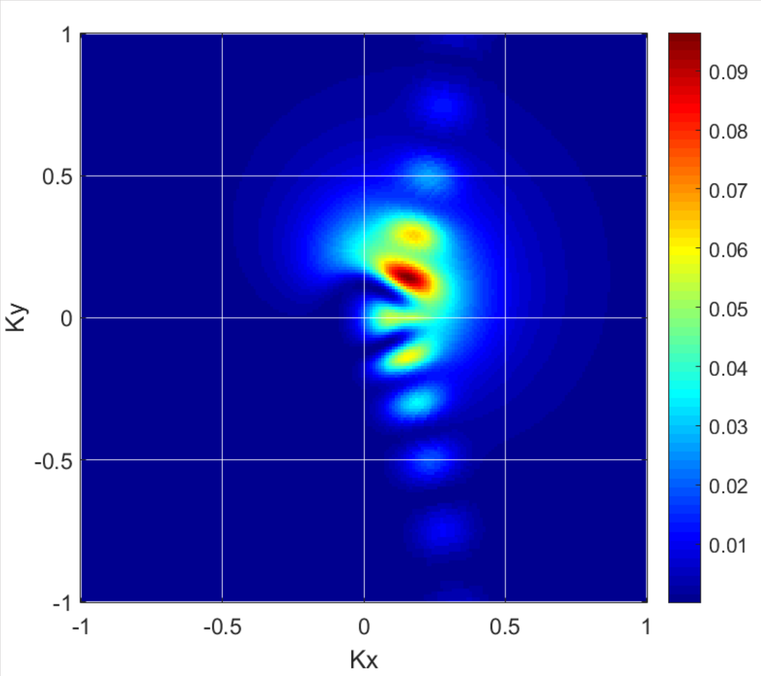
(i) Ambient wave spectrum

Established theory and practice, but parametric wave models must be used with care because of environmental nonstationarity



(ii) Wake spectrum

Hydrodynamics well understood within the framework of potential theory, as implemented under various approximations. Use of RANS and other CFD approaches enables more realistic modelling and description of turbulence

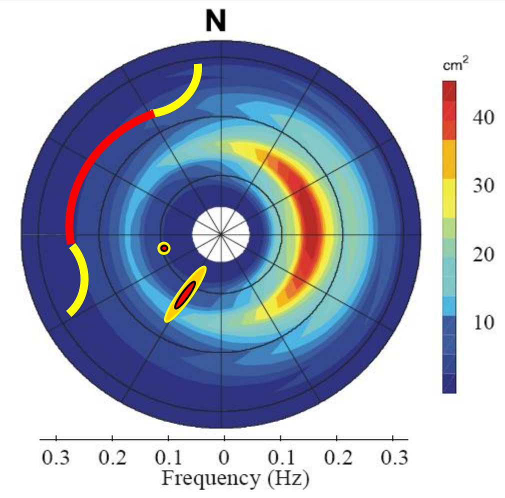


(iii) Total spectrum

Within the context of weak turbulence theory, no new physics enters but for our application we need to quantify the dissipation rate for waves on the wake manifold under the prevailing ambient wave spectrum

HF radiowave scattering from a composite wake – wave surface II

The calculation of the scattering from the composite *wave + wake* surface proceeds by assuming the statistical independence of the respective wave height contributions, analogous to the Barrick SPM2 approach, to partition the directional wave spectrum,



$$S(\vec{\kappa}) = S_{wake}(\vec{\kappa}) + S_{wave}(\vec{\kappa})$$

so we can substitute and expand the integrands in the scattering formula,

$$\widetilde{D}(\vec{k}_{scat}, \vec{k}_{inc}; \omega) = \int d\vec{\kappa}_1 F_1(\vec{k}_{scat}, \vec{k}_{inc}, \vec{\kappa}_1)[S_{wake}(\vec{\kappa}_1) + S_{wave}(\vec{\kappa}_1)]$$

$$+ \iint d\vec{\kappa}_1 d\vec{\kappa}_1 F_2(\vec{k}_{scat}, \vec{k}_{inc}, \vec{\kappa}_1, \vec{\kappa}_2)[S_{wake}(\vec{\kappa}_1) + S_{wave}(\vec{\kappa}_1)] \times [S_{wake}(\vec{\kappa}_2) + S_{wave}(\vec{\kappa}_2)]$$

HF radiowave scattering from a composite wake – wave surface III

$$\tilde{D}\left(\vec{k}_{scat},\vec{k}_{inc};\omega(\kappa)\right) = \int d\vec{\kappa}_1 F_1\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1\right)[S_{wave}(\vec{\kappa}_1)]$$

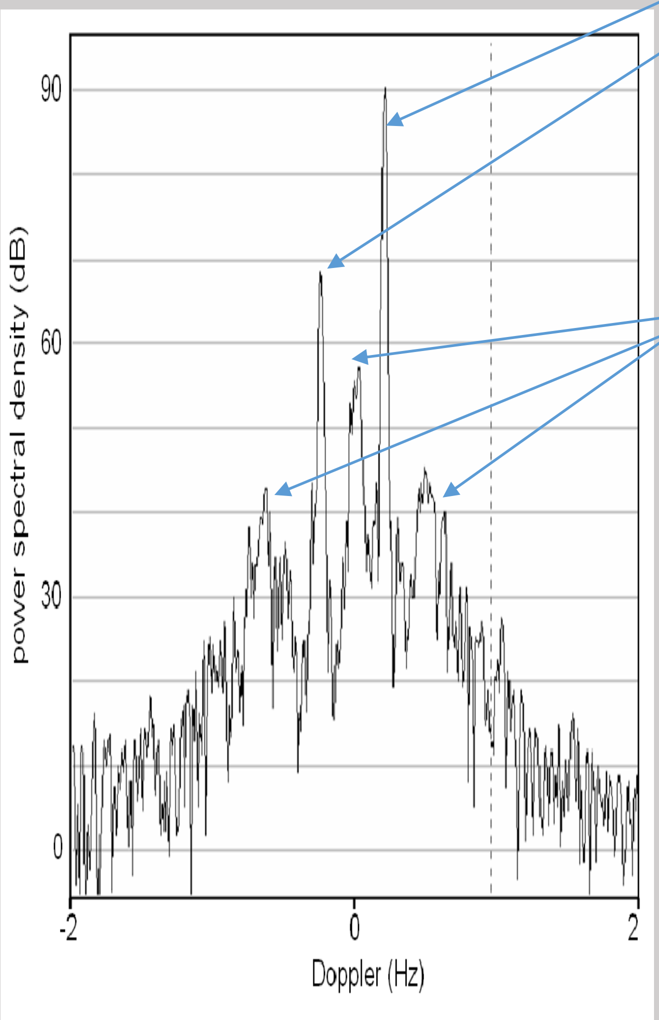
$$+\int d\vec{\kappa}_1 F_1\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1\right)[S_{wake}(\vec{\kappa}_1)]$$

$$+\iint d\vec{\kappa}_1 d\vec{\kappa}_2 F_2\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1,\vec{\kappa}_2\right)[S_{wave}(\vec{\kappa}_1)S_{wave}(\vec{\kappa}_2)]$$

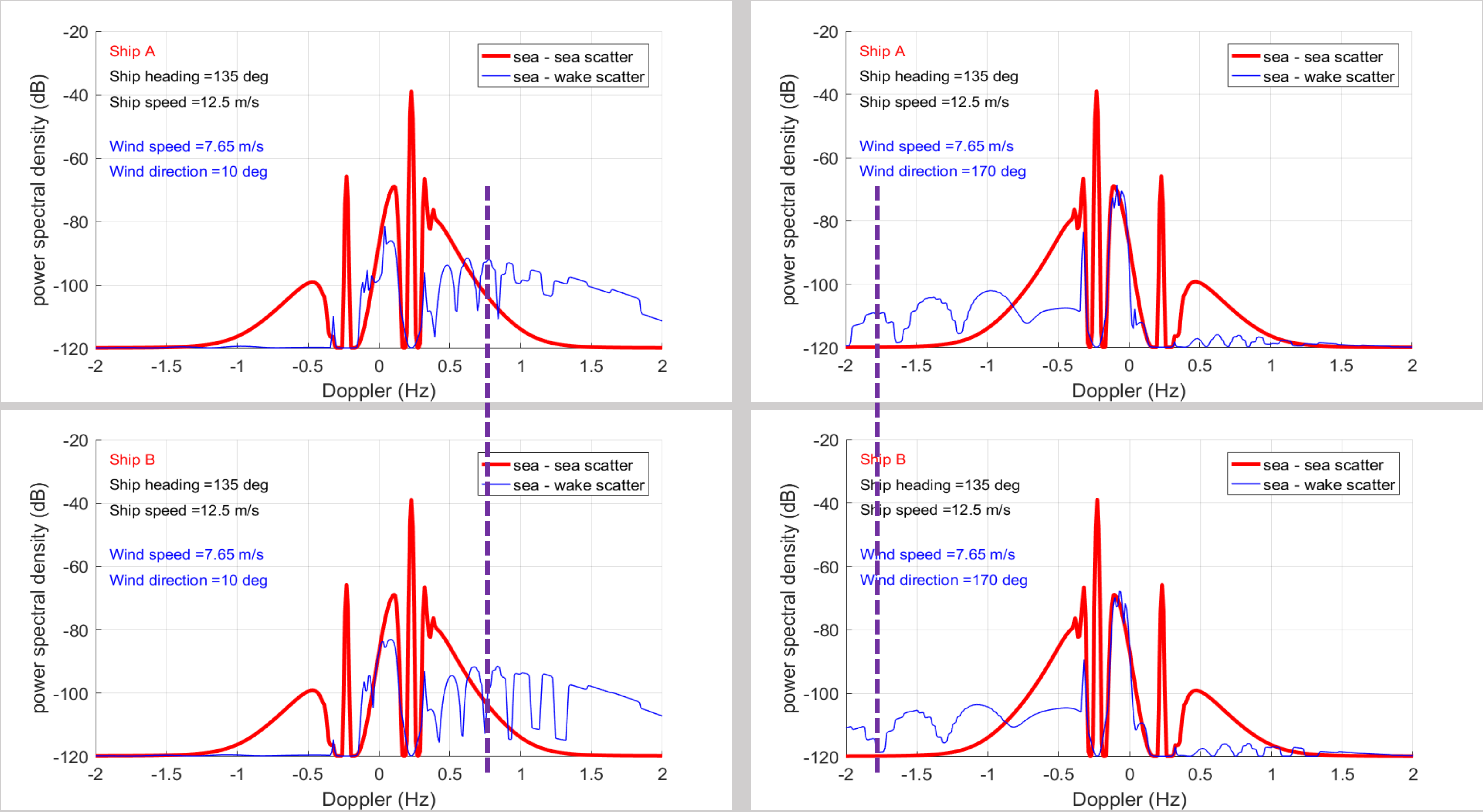
$$+\iint d\vec{\kappa}_1 d\vec{\kappa}_2 F_2\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1,\vec{\kappa}_2\right)[S_{wake}(\vec{\kappa}_1)S_{wake}(\vec{\kappa}_2)]$$

$$+\iint d\vec{\kappa}_1 d\vec{\kappa}_2 F_2\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1,\vec{\kappa}_2\right)[S_{wave}(\vec{\kappa}_1)S_{wake}(\vec{\kappa}_2)]$$

$$+\iint d\vec{\kappa}_1 d\vec{\kappa}_2 F_2\left(\vec{k}_{scat},\vec{k}_{inc},\vec{\kappa}_1,\vec{\kappa}_2\right)[S_{wake}(\vec{\kappa}_1)S_{wave}(\vec{\kappa}_2)]$$



Potential ship classification capability based on wake signature



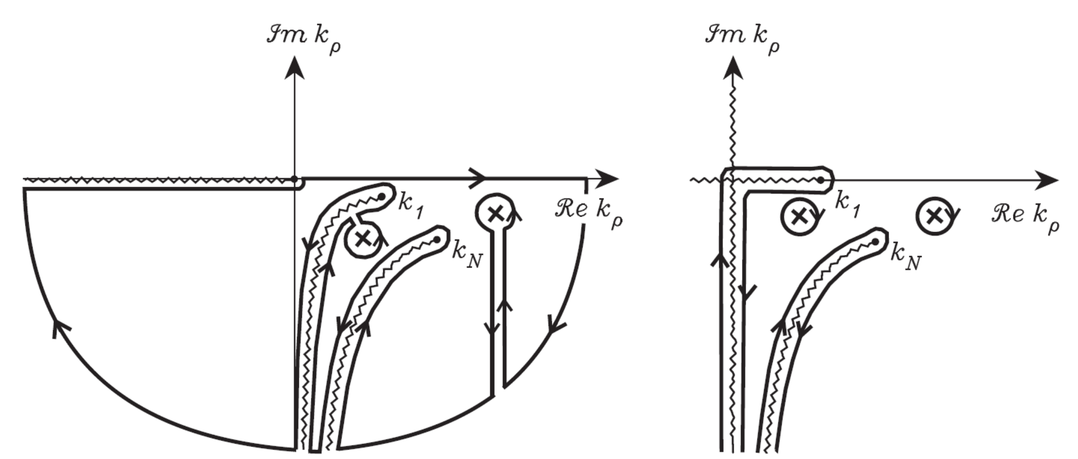
Waves in ice fields : Boundary conditions at the ocean surface

Hydrodynamic

- free surface
- two fluid, uniform shear surface
- stratified fluid rigid lid
- mass-loaded surface
- viscous layer
- viscoelastic layer
- visco-elastic-brittle layer .....

Electromagnetic

- perfect electrical conductor
- imperfect conductor
- PEC with infinitesimal dielectric coating
- PEC with finite thickness dielectric coating
- anisotropic impedance surface
- nonlinear impedance surface
-



(a) Integration path closed by a semicircular arc at infinity in the lower halfplane.

(b) Integration path wrapped around fundamental branch cuts and poles assuming a loss-less upper halfspace.

Dispersion relations for various sea ice models

Free surface $\omega^2 = g\kappa \tanh \kappa H \equiv gZ$

Mass loading $\omega^2 = \dfrac{gZ}{1 + \frac{\rho_{ice}}{\rho} hZ}$

Viscous layer $\omega^2 = \dfrac{\left(1 - \rho_{ice}\dfrac{h\omega^2}{g\rho} + \dfrac{\kappa^2 gh\left(\frac{\rho_{ice}-\rho}{\rho}\right)(\omega^2 - 4i\kappa^2\omega\nu)}{\omega^4 + 16\kappa^4\omega^2\nu^2}\right)}{\left(1 - \dfrac{\kappa^2 gh(\omega^2 - 4i\kappa^2\omega\nu)}{\omega^4 + 16\kappa^4\omega^2\nu^2}\right)} gZ$

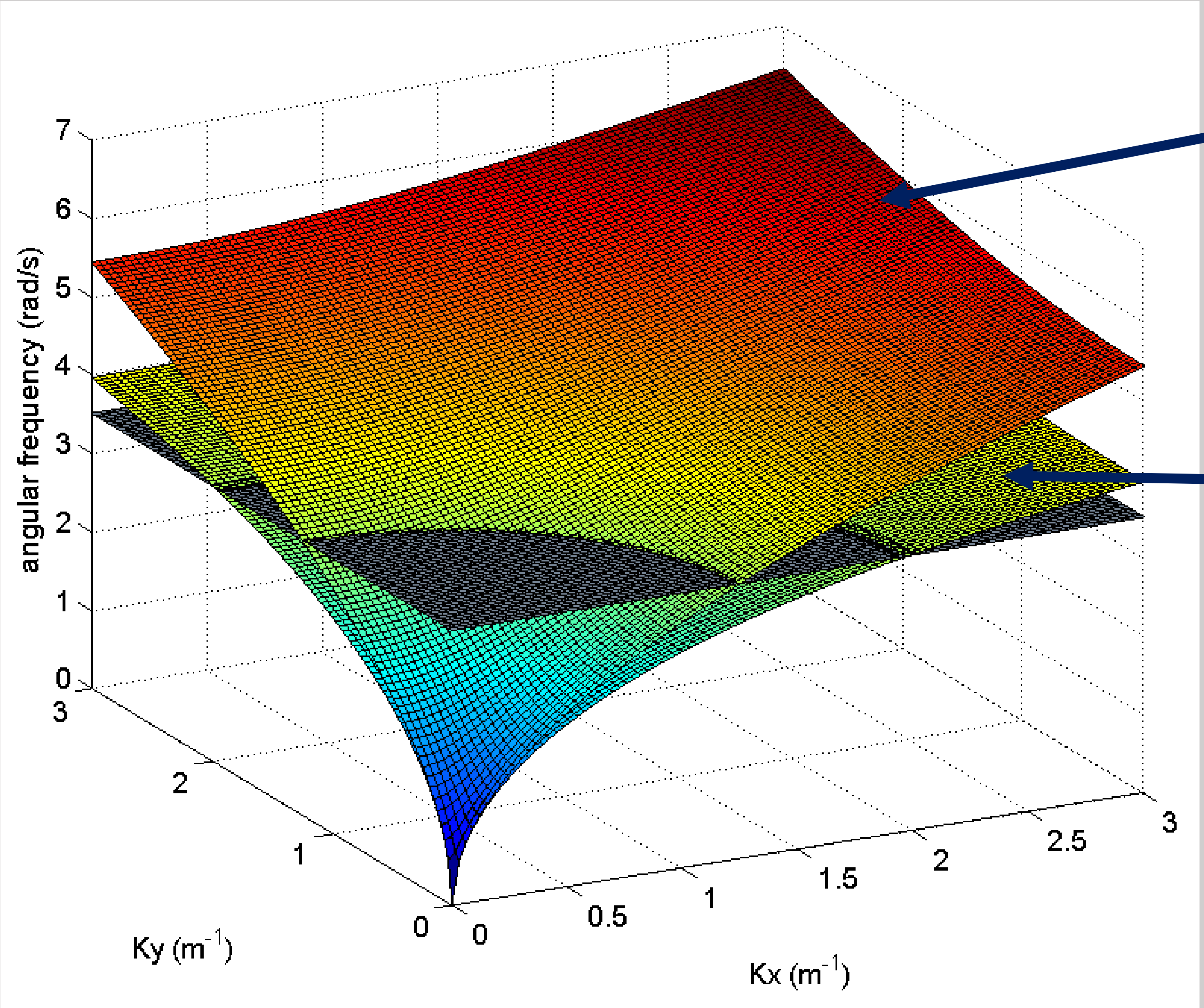
Thin elastic plate $\omega^2 = \dfrac{\left(g + \frac{L}{\rho}\kappa^4\right)Z}{1 + \frac{\rho_{ice}}{\rho} hZ}$

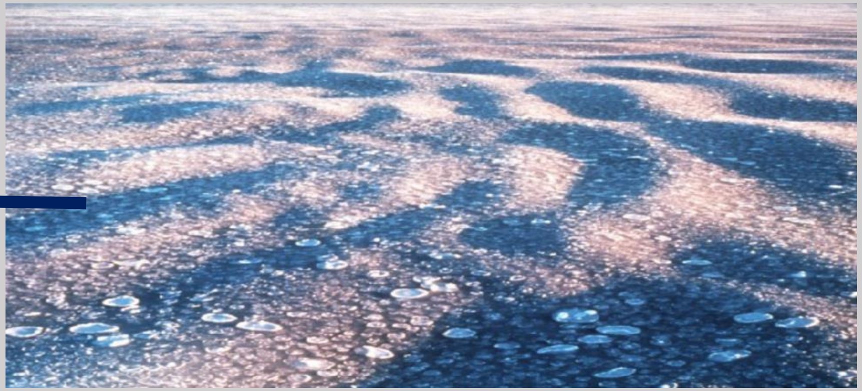
Viscoelastic layer $\omega^2 = Q\, gZ$

where $Q = 1 + \dfrac{\rho_{ice}}{\rho} \cdot \dfrac{g^2\kappa^2 S_\kappa S_\alpha - (N^4 + 16\kappa^6\alpha^2\nu_e^4)S_\kappa S_\alpha - 8\kappa^3\alpha\nu_e^2 N^2(C_\kappa C_\alpha - 1)}{g\kappa(4\kappa^3\alpha\nu_e^2 S_\kappa C_\alpha + N^2 S_\alpha C_\kappa - g\kappa S_\kappa S_\alpha)}$

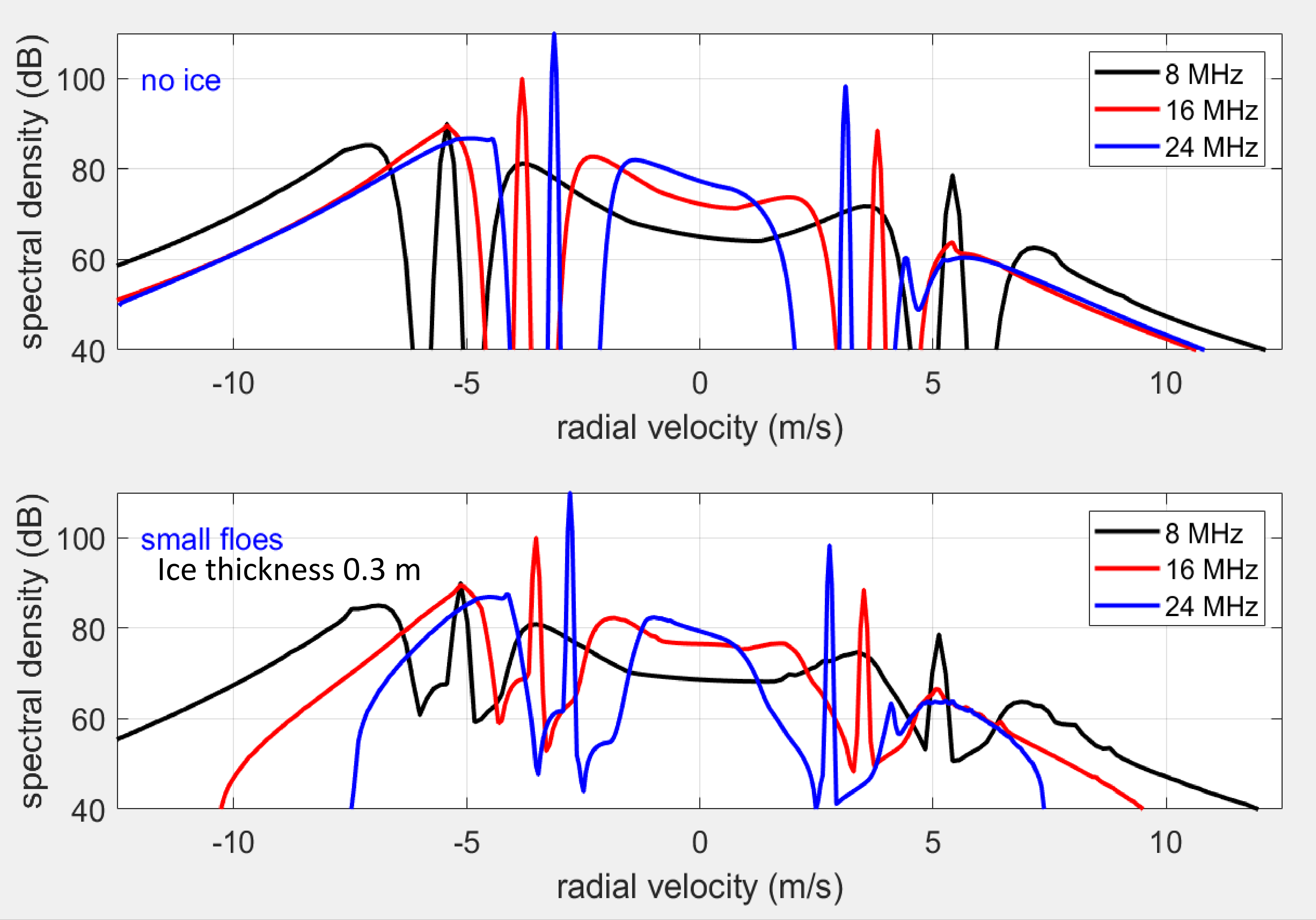
H is the water depth
h is the ice thickness
L is the flexural rigidity
$L = \dfrac{Eh^3}{12(1 - P^2)}$
G is the effective shear modulus
P is the Poisson ratio
E is Young's modulus
ν_e is the effective kinematic viscosity
$N = \omega + 2i\kappa^2\nu_e$
$\alpha^2 = \kappa^2 - i\frac{\omega}{\nu_e}$
$\nu_e = \nu + \dfrac{iG}{\rho_{ice}\omega}$

Dispersion relations for free surface and pancake ice (viscous not shown)



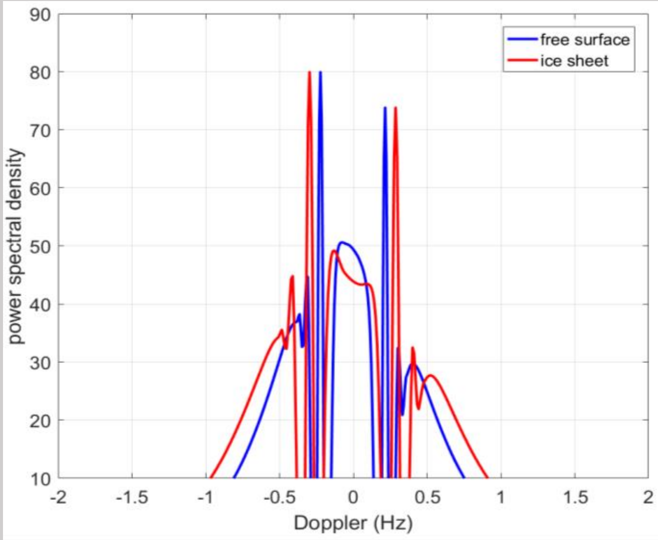


Consequences of the changed dispersion in the presence of small ice floes



Computed for monostatic scattering geometry

NB : The lower figure is not a filtered version of the upper : it shows Doppler spectra from different wave species which happen to have identical spatial spectra



When all else fails, ask a computer

Given a vast database of directional wave spectra over contiguous volumes of space and time, can a neural network :

- discover the patterns that reflect the operation of the source terms ?
- reconstruct the associated mathematical operators in forms that can be applied to wave modelling and forecasting ?
- provide a report in the language of physics that explains why the proposed functions were selected to embody the observed behavior ?

Nature 7 November 2019

NEWS · 07 NOVEMBER 2019

AI Copernicus 'discovers' that Earth orbits the Sun

A neural network that teaches itself the laws of physics could help to solve quantum-mechanics mysteries.

Davide Castelvecchi

Physicists have designed artificial intelligence that thinks like the astronomer Nicolaus Copernicus by realizing the Sun must be at the centre of the Solar System. Credit: NASA/JPL/SPL

RELATED ARTICLES

Reimagining of Schrödinger's cat breaks quantum mechanics – and stumps physicists

AI peer reviewers unleashed to ease publishing grind

Quantum machine goes in search of the Higgs boson

How machine learning could help to improve climate forecasts

Machine learning spots treasure trove of elusive viruses

Conclusion

HF radar has some unique capabilities, not all of which have been fully exploited

The task of establishing the detailed functional forms of the source terms in the Hasselmann equation (or its more general relatives) is a huge challenge – as this audience knows well

Perhaps the embryonic ideas outlined in this talk will lead to better ideas that bear fruit